

Panel 2: Drivers of Equilibrium Interest Rates¹

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1 Introduction

For the purposes of these remarks, the natural rate of interest, or r_t^* for short, is defined as the unobserved permanent component of the real short-term interest rate. This definition aligns with the one used, for example, in the seminal work by Laubach and Williams (2003). The natural rate of interest is also sometimes defined as the real short-term interest rate that would prevail in the absence of nominal rigidities. However, this is a different concept and not the one employed here. The latter measure is affected by cyclical variations in transitory shocks hitting the economy, whereas the time path of the former measure is independent of such transitory shocks.

In general, r_t^* is regarded as independent of monetary policy but at the same time of great importance to it. Given a central bank's inflation target, the nominal short-term interest rate set by the central bank, say the fed funds rate in the case of the United States, absent any cyclical factors, should be set equal to the sum of the natural rate and the inflation target. For example, if the inflation target is 2 percent and the natural rate is 1 percent then, absent any cyclical factors, the central bank interest rate should be set at 3 percent, or in the terminology of the field, the neutral (or terminal) rate should be 3 percent. Thus it is not surprising that there exists a large body of work that aims to estimate the value of the natural rate of interest.

2 Empirical Estimates of the Time Path of the Natural Rate of Interest, r_t^*

Much of the empirical literature on r_t^* has focused on estimating its path. See, for example, Laubach and Williams (2003, 2016); Del Negro et al. (2017, 2019); Holston et al. (2017); Ferreira and Shousha (2021); Cesa-Bianchi et al. (2022); and Hamilton et al. (2016).

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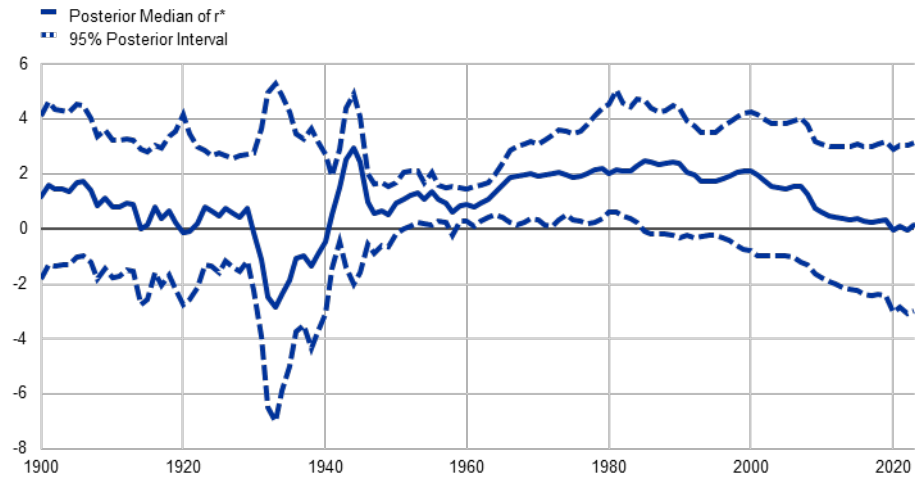
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Chart 1

Time Path of the Natural Rate of Interest (r_t^*), U.S. annual data 1900-2023

(percent per year)



Source: Schmitt-Grohé and Uribe (2024), Figure 1.

Notes: The natural rate, r_t^* , is computed by two-sided Kalman smoothing. It is normalized by adding a constant to match the observed sample mean of $i_t - \pi_{t+1}$ (1.05 percent per year). The solid line is the posterior median of r_t^* and the broken lines indicate the 2.5th and 9.75th posterior percentile of r_t^* , respectively.

In Schmitt-Grohé and Uribe (2024), using the semistructural empirical methods developed in Uribe (2022), we estimate the natural rate of interest using annual U.S. data over the period 1900 to 2023. Chart 1 shows the estimated path of the natural rate and the associated 95-percent posterior interval.

2.1 Natural Rate Supercycles

Chart 1 shows that the natural rate experiences supercycles. The first supercycle started sometime before 1900 and reached a trough around the end of the Great Depression in 1933. Then a second supercycle began and reached a peak in the early 1980s. Since that peak, the natural rate has been falling more or less monotonically at least until the onset of the pandemic in 2020. Since then, r_t^* is estimated to be relatively stable. This stability could indicate either a temporary pause in the downward part of the ongoing supercycle, or it could indicate a turning point so that a new period of rising natural rates is about to begin.

The chart further shows that the steepest declines in the estimated path of r_t^* occurred during the financial crises of 1929 and 2008. We will revisit this observation below when discussing potential theoretical explanations for variations in r_t^* .

2.2 Demographics and r^*

Some have argued that as the population ages, the rate of innovation declines, and as a consequence r^* falls. Combining the idea that population aging is a key driver of lower natural rates with the empirical regularity that in the United States population aging has steadily increased since 1900, we should see that the natural rate has

steadily decreased since 1900. But this prediction is not supported by the data, as we have just documented that the natural rate experienced two supercycles over this period. In particular, the natural rate was as low in 2023 as it was in the mid-1920s.

For the purposes of the present remarks, the main takeaway from Chart 1 is that r_t^* displays supercycles. We will refer to this empirical property as stylized fact 1.

Stylized Fact 1: The natural rate of interest, r_t^* , displays supercycles.

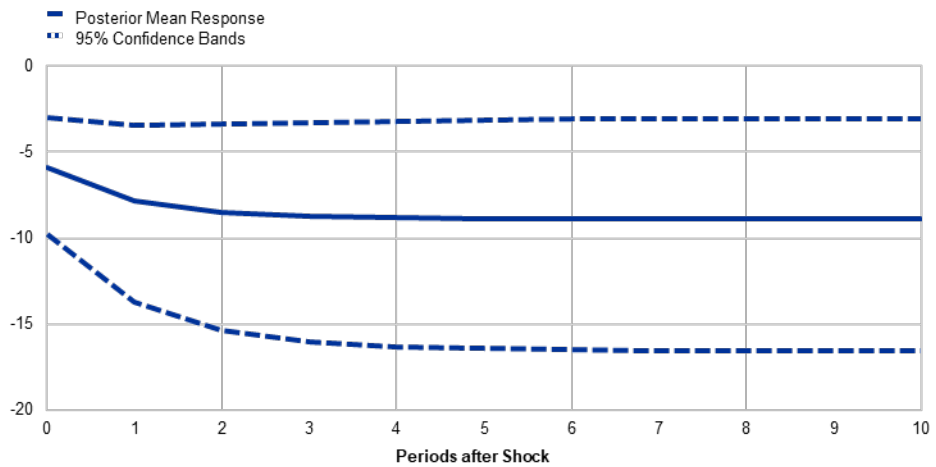
3 Empirical Estimates of the Consequences of Natural Rate Shocks

Chart 1 shows the estimated path of the natural rate of interest—a topic on which a large literature exists, as mentioned above. Less research exists on the empirical question of how movements in the natural rate affect macroeconomic indicators in the short and long runs.

Chart 2

Impulse Response to a 1% Decline in the Natural Rate of Interest, r_t^*

(percent, deviation from pre-shock level)



Source: Schmitt-Grohé and Uribe (2024). Figure 3.

Notes: Solid lines display the posterior mean response to a negative natural rate shock (a decrease in r_t^*) that lowers the real interest rate by 1 percentage point in the long run. Broken lines are asymmetric 95-percent confidence bands computed using the method of Sims and Zha (1999).

Chart 2, also taken from Schmitt-Grohé and Uribe (2024), addresses this question. It displays the posterior mean response to a negative natural rate shock (a fall in r_t^*) that lowers the real short-term interest rate by 1 percentage point in the long run. The chart includes asymmetric 95-percent confidence bands computed using the Sims-Zha (1999) method. The estimates indicate that a fall in the natural rate decreases the trend level of output. Specifically, a one percentage point decline in the natural rate is estimated to lower the trend component of the level of real GDP per capita between 3 and 16 percent, with a mean of 9 percent. This result is our second stylized fact.

Stylized Fact 2: A decline in the natural rate of interest, r_t^* , lowers the trend level of real GDP per capita.

Given our findings, namely, that the natural rate displays supercycles and that a negative shock to the natural rate shock shifts the trend growth path of the natural logarithm of real output per capita down in a parallel fashion, we next ask what theories are consistent with these two stylized facts.

4 Theory: Drivers of r^*

The results of the preceding analysis delivered two stylized facts about r^* : (i) over the past 124 years r^* has displayed supercycles and (ii) a shock that lowers r^* leads to a sizeable downward shift in the level of the trend growth path of the natural logarithm of real GDP per capita.

Here we first show that these two stylized facts are difficult to reconcile with the predictions of a canonical neoclassical growth model. A key challenge is to explain stylized fact 2, namely, that a fall in r^* is associated with a decline in the trend level of real per capita output.

Motivated by the above observation that in the 1900 to 2023 sample, the sharpest declines in the estimated path of r_t^* occurred during the financial crises of 1929 and 2008, we then embed a liquidity friction into the canonical neoclassical growth model. The resulting model predicts that along the balanced growth path a decline in liquidity drives down both r^* and the level of trend output, which is consistent with stylized fact 2.

4.1 A Neoclassical Growth Model

Consider the canonical neoclassical growth model with exogenous population growth and labor augmenting technological change. Let N_t denote the number of workers and assume that $N_{t+1} = (1 + n)N_t$, where n denotes the rate of population growth. Let A_t denote the level of labor augmenting technology and assume that $A_{t+1} = (1 + g)A_t$, where g denotes the growth rate of labor augmenting technological change. Define r^* as the real interest rate along the balanced growth path. As shown in Appendix A, along the balanced growth path,

$$r^* = \frac{(1 + g)^\sigma}{\beta} - 1. \quad (1)$$

This means that the only parameters affecting r^* are the growth rate of labor augmenting technical change, g , the household's coefficient of relative risk aversion, $\sigma > 0$, and the household's subjective discount factor, $\beta \in (0, 1)$. Importantly, r^* is independent of the population growth rate n . As such the neoclassical growth model cannot be used to argue that a slowdown in population growth is driving the decline in r^* .

Along the balanced growth path, the natural logarithm of real GDP per capita, denoted $\ln Y_t/N_t$, is given by (see Appendix A for details),

$$\ln\left(\frac{Y_t}{N_t}\right) = \frac{\alpha}{1-\alpha} [\ln \alpha - \ln(r^* + \delta)] + \ln A_t, \quad (2)$$

where $(1 - \alpha) \in (0, 1)$ denotes the labor share and δ denotes the rate of depreciation of physical capital. This expression shows that holding constant the rate of technological progress, g , there is an inverse relationship between r^* and real GDP per capita along the balanced growth path. Thus, holding constant g , the predicted relationship between r^* and the trend level of per capita real GDP is not consistent with stylized fact 2.

What about changes in the rate of technological progress g ? By (1), the neoclassical model predicts that the higher g is, the larger r^* will be. This suggests that in the neoclassical growth model g could be an important driver of r^* . For example, if one were to interpret the recent advances in generative AI as ushering in a period of persistently high technological growth, that is, an increase in g , then based on this observation, one might expect them to be associated with r^* rising. One potential objection to this argument is that in the data, the degrees of integration of the growth rate of real GDP per capita and the real short-term interest rate are different, while in the neoclassical growth model they are assumed to be the same. In the data real per-capita output growth is stationary, that is, integrated of order 0, whereas the real interest rate is nonstationary, and integrated of order 1. These considerations cast doubt on the prediction of the neoclassical growth model that technological change is an important driver of r^* .

Overall, the arguments presented suggest that to understand the drives of r^* one might have to look beyond the neoclassical growth model.

4.2 A Neoclassical Growth Model with a Liquidity Friction

In this section, we introduce a financial or liquidity friction into the neoclassical growth model presented above. For simplicity we abstract from population growth and technological progress.

Households are assumed to be subject to a working capital constraint for investment, denoted i_t . At the beginning of the period, prior to production taking place, households make investment decisions that determine next period's capital stock, k_{t+1} .⁴ The working capital constraint requires that period- t investment must be less than or equal to a function of the household's assets at the beginning of period t . These assets consist of the maturing bonds acquired in the previous period, b_t , and

⁴ Because the model abstracts from population and technology growth, all variables are stationary and accordingly we use lower case letters to denote them.

the capital stock, k_t . Specifically, assume that the working capital constraint on investment takes the form

$$i_t \leq \rho b_t^\gamma k_t^{1-\gamma}, \quad \gamma \in [0, 1),$$

and $\rho > 0$ is a parameter controlling the severity of the constraint. For a sufficiently low supply of bonds, b , there exists a balanced growth path in which the working capital constraint is binding.⁵ Along that balanced growth path, an exogenous decline in the quantity of liquidity, b , lowers both the natural rate of interest, r^* , and the trend level of per capita output, $y = Y/N$, so that the model is consistent with stylized fact 2.

The mechanism in the model works as follows. Consider the balanced growth path. Both capital, k , and liquidity, b , relax the investment constraint. Capital is an endogenous variable. The steady state value of k is increasing in the amount of liquidity b , because more b allows for higher steady state investment and hence higher steady state k . With higher steady state capital, output in the steady state will also be higher. This establishes a positive relationship between steady-state liquidity b and steady-state per capita output y .

The reason why the model also predicts a positive relationship between r^* and liquidity in the steady state is that a bond provides two benefits to its holder. One benefit is the interest the bond pays, r^* . The second benefit the bond has is that it relaxes the investment constraint. Thus, when the shadow price of investment falls (in the sense that the investment constraint while still binding is less restrictive), the second benefit provided by the bond is smaller and thus households require more compensation in the form of interest payments (r^*) to be willing to hold a given quantity of bonds. This implies that r^* is increasing in b .

Taken together, we have that, provided the investment constraint is binding, a negative shock to liquidity, b , lowers the level of per capita output and at the same time lowers r^* , consistent with stylized fact 2.

Appendix

Appendix A: A Neoclassical Growth Model

Appendix A derives equation (1) and (2) of section 4.1 of the main text.

The production function is assumed to take the form

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \quad \alpha \in (0, 1),$$

where Y_t denotes output, K_t denotes the capital stock, and α is a parameter. The capital stock evolves over time according to

⁵ For details, see Appendix B.

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where δ denotes the depreciation rate and I_t denotes investment.

Households have preferences over per capita consumption, C_t/N_t

$$\sum_{t=0}^{\infty} \beta^t N_t U\left(\frac{C_t}{N_t}\right).$$

The period felicity function takes the form: $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. For simplicity, we assume that each worker supplies inelastically 1 unit of labor. Households are assumed to have access to a one-period bond, denoted b_t , that pays the real interest rate r_t when held from period t to period $t + 1$ and pay lump-sum taxes T_t in period t to the government. The sequential budget constraint of the household can then be expressed as

$$C_t + K_{t+1} - (1 - \delta)K_t + \frac{B_t}{1 + r_t} = K_t^\alpha (A_t N_t)^{1-\alpha} + B_t - T_t.$$

Each period t , the government issues discount bonds, $B_{t+1}/(1 + r_t)$, and collects lump-sum taxes T_t . Its sequential budget constraint is given by $B_{t+1}/(1 + r_t) = B_t + T_t$ for all $t \geq 0$, with B_0 given.

Without loss of generality assume that bonds are in zero net supply, including in period 0, $B_0 = 0$. An equilibrium then are sequences for Y_t, K_{t+1}, C_t, I_t , and r_t satisfying

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha} \tag{A1}$$

$$C_t + I_t = Y_t \tag{A2}$$

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{A3}$$

$$\left(\frac{\frac{C_{t+1}}{N_{t+1}}}{\frac{C_t}{N_t}}\right)^\sigma = \beta(1 + r_t) \tag{A4}$$

$$\left(\frac{\frac{C_{t+1}}{N_{t+1}}}{\frac{C_t}{N_t}}\right)^\sigma = \beta[\alpha K_{t+1}^{\alpha-1} (A_{t+1} N_{t+1})^{1-\alpha} + 1 - \delta] \tag{A5}$$

given initial K_{-1} and exogenous sequences for N_t and A_t . Because there is labor augmenting technological progress and population growth some variables are non-stationary. We perform the following stationarity inducing transformations: $x_t = X_t/(A_t N_t)$, for $X = Y, K, C$, and I . We can then write the equilibrium conditions as

$$y_t = k_t^\alpha \quad (\text{A6})$$

$$c_t + i_t = y_t \quad (\text{A7})$$

$$(1 + g)(1 + n)k_{t+1} = (1 - \delta)k_t + i_t \quad (\text{A8})$$

$$\left(\frac{(1 + g)c_{t+1}}{c_t} \right)^\sigma = \beta(1 + r_t) \quad (\text{A9})$$

$$1 + r_t = \alpha k_{t+1}^{\alpha-1} + 1 - \delta. \quad (\text{A10})$$

Consider next the balanced growth path of this economy. This is of interest because r^* is defined as the real interest rate along the balanced growth path. A balanced growth path is an equilibrium in which all stationary variables are constant over time, $x_t = x_{ss}$, for $x = y, k, c, i, r$. The equations describing the balanced growth path then are

$$y_{ss} = k_{ss}^\alpha \quad (\text{A11})$$

$$c_{ss} + i_{ss} = y_{ss} \quad (\text{A12})$$

$$[(1 + g)(1 + n) - (1 - \delta)]k_{ss} = i_{ss} \quad (\text{A13})$$

$$(1 + g)^\sigma = \beta(1 + r_{ss}) \quad (\text{A14})$$

$$1 + r_{ss} = \alpha k_{ss}^{\alpha-1} + 1 - \delta. \quad (\text{A15})$$

Because r^* is defined as the real interest rate along the balanced growth path, we have

$$r^* = r_{ss}.$$

Equation (1), then follows immediately from (A14).

With r^* in hand and knowing in addition α and δ , equation (A15) gives $k_{ss} = \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{1}{1-\alpha}}$. Because $\alpha \in (0,1)$, this expression says that a decline in r^* raises the value of k_{ss} . In turn, the value of y_{ss} , by equation (A11), is equal to $y_{ss} = k_{ss}^\alpha = \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}}$, which shows that along the balanced growth path r^* and y_{ss} are inversely related. Combining the definition $y_t \equiv Y_t/(A_t N_t)$ with the definition of real GDP per capita, we have $Y_t/N_t = y_t A_t$. The natural logarithm of real GDP per capita along the balanced growth path can therefore be expressed as $\ln \left(\frac{Y_t}{N_t} \right) = \ln(y_{ss}) + \ln A_t$. Finally, replacing y_{ss} with the above expression for y_{ss} , we obtain equation (2).

Appendix B

The representative household maximizes $\sum_{t=0}^{\infty} \beta^t U(c_t)$ subject to the sequential budget constraint $c_t + k_{t+1} - (1 - \delta)k_t + \frac{b_{t+1}}{1+r_t} = k_t^\alpha + b_t - \tau_t$ where τ_t denotes lump-sum taxes paid by the household in period t , and the liquidity friction $k_t - (1 - \delta)k_t \leq \kappa b_t^\gamma k_t^{1-\gamma}$, given initial conditions k_0 and b_0 . The associated first-order conditions are $U'(c_t) = \lambda_t$, $\frac{\lambda_t}{1+r_t} = \beta \lambda_{t+1} [1 + \mu_{t+1} \kappa \gamma b_{t+1}^{\gamma-1} k_{t+1}^{1-\gamma}]$, and $\lambda_t (1 + \mu_t) = \beta \lambda_{t+1} \{\alpha k_{t+1}^{\alpha-1} + 1 - \delta + \mu_{t+1} [\kappa b_{t+1}^\gamma (1 - \gamma) k_{t+1}^{-\gamma} + 1 - \delta]\}$. The government's budget constraint is $\frac{b_{t+1}}{1+r_t} + \tau_t = b_t$. By market clearing $c_t + k_{t+1} - (1 - \delta)k_t = k_t^\alpha$. An equilibrium then are sequences for $\{c_t, k_t, \lambda_t, \mu_t, r_t\}$ satisfying

$$c_t + k_{t+1} - (1 - \delta)k_t = k_t^\alpha \quad (\text{B16})$$

$$U'(c_t) = \lambda_t \quad (\text{B17})$$

$$k_t - (1 - \delta)k_t \leq \kappa b_t^\gamma k_t^{1-\gamma}; \mu_t \geq 0; \mu_t [\kappa b_t^\gamma k_t^{1-\gamma} - (k_{t+1} - (1 - \delta)k_t)] = 0 \quad (\text{B18})$$

$$\frac{\lambda_t}{1+r_t} = \beta \lambda_{t+1} [1 + \mu_{t+1} \kappa \gamma b_{t+1}^{\gamma-1} k_{t+1}^{1-\gamma}] \quad (\text{B19})$$

$$\lambda_t (1 + \mu_t) = \beta \lambda_{t+1} \{\alpha k_{t+1}^{\alpha-1} + 1 - \delta + \mu_{t+1} [\kappa b_{t+1}^\gamma (1 - \gamma) k_{t+1}^{-\gamma} + 1 - \delta]\} \quad (\text{B20})$$

given an exogenous supply of bonds, b_t , and the initial capital stock k_0 .

In a steady state by definition $x_t = x$ for all t and for $x = c, k, \mu, \lambda, r$, given a constant value of bonds, $b_t = b$. Evaluate (B16), (B17), (B18), (B19), and (B20) at the steady state. This yields:

$$c + \delta k = k^\alpha \quad (\text{B21})$$

$$U'(c) = \lambda \quad (\text{B22})$$

$$\delta k \leq \kappa b^\gamma k^{1-\gamma}; \quad \mu \geq 0; \quad \mu[\kappa b^\gamma k^{1-\gamma} - \delta k] = 0 \quad (\text{B23})$$

$$\frac{1}{1+r} = \beta[1 + \mu \kappa \gamma b^{\gamma-1} k^{1-\gamma}] \quad (\text{B24})$$

$$1 + \mu = \beta\{\alpha k^{\alpha-1} + 1 - \delta + \mu[\kappa b^\gamma (1-\gamma)k^{-\gamma} + 1 - \delta]\}. \quad (\text{B25})$$

Note that given steady-state values for r , μ and k , the steady-state values of c and λ can be read off from equations (B21) and (B22), respectively. Thus, in what follows we will limit attention to steady state conditions (B23), (B24), and (B25). Suppose the liquidity constraint is not binding, $\mu = 0$. Denote the steady-state values associated with this case with a u (for unconstrained) superscript. Setting $\mu = 0$ in (B24) gives $1 + r^u = 1/\beta$. With r^u in hand, we can find k^u as the solution to (B25), $1 + r^u = \alpha k^{u\alpha-1} + 1 - \delta$. Evaluating (B23) at these values for r^u and k^u , we find that the unconstrained steady state only exists for a supply of the bond b in excess of the lower bound \bar{b} , that is, when $b \geq \bar{b} \equiv \left(\frac{\delta}{\kappa}\right)^{\frac{1}{\gamma}} k^u$. Assume now that $b < \bar{b}$. In this case, as we have just shown, $\mu = 0$ cannot be supported as a steady state. We wish to show that a steady state exists in which $\mu > 0$. If $\mu > 0$, then by (B23), it must be that

$$\frac{b}{k} = \left(\frac{\delta}{\kappa}\right)^{\frac{1}{\gamma}}. \quad (\text{B26})$$

Solve (B25) for μ and use the above expression to eliminate b/k . This yields

$$\mu = \frac{\beta(\alpha k^{\alpha-1} + 1 - \delta) - 1}{1 - \beta[(1-\gamma)\delta + (1-\delta)]}. \quad (\text{B27})$$

Notice that the denominator of the fraction on the right-hand side of (B27) is positive as γ, δ and β are positive and less than one. To find the sign of the numerator, notice that at $k = k^u$, the numerator is zero. If $k < k^u$, then the numerator is positive, and hence $\mu > 0$. But if $k > k^u$, then the numerator is negative and μ would be negative. Thus, if a steady state exists with $\mu > 0$, it must be the case that $k < k^u$. Then by (B26) we find that $b = \left(\frac{\delta}{\kappa}\right)^{1/\gamma} k < \left(\frac{\delta}{\kappa}\right)^{1/\gamma} k^u = \bar{b}$. This means that a steady state with $\mu > 0$ exists provided $b < \bar{b}$. Finally, choose r to satisfy (B24). It follows that $r < r^u$.

In summary, when $b < \bar{b}$, a steady state exists and has the property that $r < r^u, k < k^u, \mu > 0$, and $y < y^u$.

Next compare the steady states for two values of b , denoted $b' < \bar{b}$ and $b'' < b'$. Denote the associated steady state values of output and the interest rate as y' and r' and y'' and r'' , respectively. Clearly by (B26) $k'' < k' < k^u$ and hence $y'' < y' < y^u$ and by (B27) $\mu'' > \mu' > 0$, so that from (B24) $r'' < r' < r^u$. This shows that if b falls, then so do the steady-state values of output per capita and the real interest rate, which is what we had set out to show.

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