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Estimation of linear dynamic panel data models with time-invariant regressors

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Abstract

We propose a two-stage estimation procedure to identify the effects of time-invariant regressors in a dynamic version of the Hausman-Taylor model. We first estimate the coefficients of the time-varying regressors and subsequently regress the first-stage residuals on the time-invariant regressors providing analytical standard error adjustments for the second-stage coefficients. The two-stage approach is more robust against misspecification than GMM estimators that obtain all parameter estimates simultaneously. In addition, it allows exploiting advantages of estimators relying on transformations to eliminate the unit-specific heterogeneity. We analytically demonstrate under which conditions the one-stage and two-stage GMM estimators are equivalent. Monte Carlo results highlight the advantages of the two-stage approach in finite samples. Finally, the approach is illustrated with the estimation of a dynamic gravity equation for U.S. outward foreign direct investment.

Keywords: Dynamic panel data; Time-invariant variables; Two-stage estimation; System GMM; Dynamic gravity equation

JEL Classification: C13; C23; F23

Non-technical summary

Panel data comprises of cross-sectional units, e.g. countries, firms, households, or individuals, observed at different points in time. The combination of cross-sectional and time series data allows for richer econometric model specifications and more accurate conclusions. In addition, dynamic adjustment processes can be analyzed for a broad base of cross-sectional units. In a dynamic model past observations of the variable of interest can influence the current value. Macroeconomic output growth regressions and microeconomic wage regressions are examples where dynamic panel data models are used to account for the persistence of the dependent variable.

This paper analyzes the identification of effects of time-invariant regressors in dynamic panel data models as the methods currently used can be very imprecise or are not able to handle these regressors. Time-invariant regressors play an important role in many empirical applications but estimation of the effects is non-trivial because there are various statistical problems that may arise. We discuss the existing possibilities to estimate dynamic panel data models with time-invariant explanatory variables and we propose an alternative two-stage estimation procedure. A major advantage of the two-stage approach is that misspecified assumptions on the time-invariant regressors do not influence the estimation results for the coefficients of time-varying variables. In extensive simulation studies we show that the currently most widely used estimation method, the generalized method of moments, can be quite biased whereas our method provides more precise and robust results. Furthermore, we develop a correction term for the standard errors of the second stage. Neglecting the correction term can generate misleading implications.

To illustrate these methods we estimate a dynamic gravity model to explain real bilateral outward stocks of FDI for the United States. The data set was previously used by other authors to demonstrate instrumental variable methods for static panel models with time-invariant regressors. In this case, the time-invariant variable of interest is geographical distance. We highlight the relevance of a dynamic model specification, the benefits of the proposed two-stage approach, and the importance of adequately correcting the standard errors.

1 Introduction

This paper considers estimation methods and inference for linear dynamic panel data models with a short time dimension. In particular, we focus on the identification of coefficients of time-invariant variables in the presence of unobserved unit-specific effects. In many empirical applications time-invariant variables play an important role in structural equations. In labor economics researchers are interested in the effects of education, gender, nationality, ethnic and religious background, or other time-invariant characteristics on the evolution of wages but would still like to control for unobserved time-invariant individual-specific effects such as worker's ability. As a recent example, Andini (2013) estimates a dynamic version of the Mincer equation controlling for a rich set of time-invariant characteristics. In macroeconomic cross-country studies institutional features or group-level effects play a role in explaining economic development. For example, Hoeffler (2002) studies the growth performance of Sub-Saharan Africa countries by introducing a regional dummy variable in her dynamic panel data model. Cinyabuguma and Putterman (2011) focus on within Sub-Saharan differences by adding socio-economic and geographic factors to the analysis. The analysis of bilateral trade or foreign direct investment (FDI) determinants is often based on gravity models with geographical distance as a key time-invariant factor. To account for the persistence of trade flows or FDI, Kimura and Todo (2010), Olivero and Yotov (2012), and Kahouli and Maktouf (2014) set up dynamic gravity equations.

If there is unobserved unit-specific heterogeneity, it is often hard to disentangle the effects of the observed and the unobserved time-invariant heterogeneity. Standard fixed and random effects estimators cannot be used because of multicollinearity problems and, when the time dimension is short, the familiar Nickell (1981) bias in dynamic panel data models. Therefore, it is common practice in empirical work to apply the generalized method of moments (GMM) framework proposed by Arellano and Bond (1991), Arellano and Bover (1995), and Blundell and Bond (1998), amongst others. However, as Binder et al. (2005) and Bun and Windmeijer (2010) emphasize, GMM estimators might suffer from a weak instruments problem when the autoregressive parameter approaches unity or when the variance of the unobserved unit-specific effects is large. Moreover, the number of instruments can rapidly become large relative to the sample size. The consequences of instrument proliferation, summarized by Roodman (2009), range from biased coefficient and standard error estimates to weakened specification tests.

In order to overcome the weak instruments problem in the context of estimating the effects of time-varying regressors, Hsiao et al. (2002) propose a transformed likelihood approach that is based on the model in first differences. A shortcoming of this approach is the inability to estimate the coefficients of time-invariant regressors. In this paper, we propose a two-stage estimation procedure to identify the latter. In the first stage, we estimate the coefficients of the time-varying regressors. Subsequently, we regress the first-stage residuals on the time-invariant regressors.[1] We achieve identification by using instrumental variables in the spirit of Hausman and Taylor (1981), and adjust the second-stage standard errors to account for the first-stage estimation error. Our methodology applies to any first-stage estimator that consistently estimates the coefficients of the time-varying variables without relying on coefficient estimates for the time-invariant regressors. Among others, the quasi-maximum likelihood (QML) estimator of Hsiao et al. (2002) as well as GMM estimators qualify as potential first-stage candidates. A major advantage of the two-stage approach is the invariance of the first-stage estimates to misspecifications regarding the model assumptions on the correlation between the time-invariant regressors and the unobserved unit-specific effects.[2] However, under particular conditions feasible efficient one-stage and two-stage GMM estimation are shown to be (asymptotically) equivalent.

We perform Monte Carlo experiments to evaluate the finite sample performance in terms of bias, root mean square error (RMSE), and size statistics of our two-stage procedure relative to GMM estimators that obtain all coefficient estimates simultaneously. The results suggest that the two-stage approach is to be preferred when the researcher is interested in the coefficients of both time-varying and time-invariant variables. However, the quality of the second-stage estimates depends crucially on the precision of the first-stage estimates. Among our first-stage candidates the QML estimator performs very well. GMM estimators can be an alternative if effective measures are taken to avoid instrument proliferation. Our Monte Carlo analysis unveils sizable finite sample

[1]For a static model, Plümper and Troeger (2007) propose a similar three-stage approach that they label fixed effects vector decomposition. Their first stage is a classical fixed effects regression. In a recent symposium on this method, Breusch et al. (2011) and Greene (2011) show that the first two stages can be characterized by an instrumental variable estimation with a particular choice of instruments, and that the third stage is essentially meaningless.

[2]Hoeffler (2002) and Cinyabuguma and Putterman (2011) argue similarly. They apply GMM estimation in the first stage, and ordinary least squares estimation in the second stage. However, they do not correct the second-stage standard errors.

biases when the GMM instruments are based on the full set of available moment conditions, in particular regarding the coefficients of time-invariant regressors. Finally, in contrast to conventionally computed standard errors our adjusted second-stage standard errors account remarkably well for the first-stage estimation error.

To illustrate these methods we estimate a dynamic gravity equation for FDI based on U.S. data previously employed by Egger and Pfaffermayr (2004a). We find strong evidence for history dependence of the real bilateral stock of outward FDI. Neglecting the dynamic nature of the model results in a sizable overestimation of the effect of the time-invariant geographical distance variable. Again, the correct adjustment of the second-stage standard errors proves to be important for valid inference.

The paper is organized as follows: Section 2 introduces the dynamic Hausman and Taylor (1981) model. Section 3 describes one-stage GMM estimators that identify all coefficients simultaneously, while Section 4 lays out the two-stage procedure that yields sequential coefficient estimates. Section 5 contrasts the two approaches on theoretical grounds, while Section 7 provides simulation evidence on the performance of the two-stage approach in comparison to one-stage GMM estimators under different scenarios. In Section 8 we discuss the empirical application, and Section 9 concludes.

2 Model

Consider the dynamic panel data model with units $i = 1, 2, \dots, N$, and a fixed number of time periods $t = 1, 2, \dots, T$, with $T \geq 2$:

$$y_{it} = \lambda y_{i,t-1} + \mathbf{x}_{it}'\boldsymbol{\beta} + \mathbf{f}_i'\boldsymbol{\gamma} + e_{it}, \quad e_{it} = \alpha_i + u_{it}, \tag{1}$$

where $\mathbf{x}_{it}$ is a $K_x \times 1$ vector of time-varying variables. The initial observations of the dependent variable, y_{i0}, and the regressors, $\mathbf{x}_{i0}$, are assumed to be observed. $\mathbf{f}_i$ is a $K_f \times 1$ vector of observed time-invariant variables that includes an overall regression constant, and α_i is an unobserved unit-specific effect of the i-th cross section. In a strict sense, α_i is called a fixed effect if it is allowed to be correlated with all of the regressor variables $\mathbf{x}_{it}$ and $\mathbf{f}_i$, and it is a random effect if it is independently distributed. Note that α_i is correlated with the lagged dependent variable by construction. In this paper we look at a hybrid (or intermediate case) of the dynamic fixed and random effects models where some of the regressors are correlated with α_i but not all of them. Throughout the paper we maintain the following assumptions:

Assumption 1: The disturbances u_{it} and the unobserved unit-specific effects α_i are independently distributed across i and satisfy $E[u_{it}] = E[\alpha_i] = 0$, $E[u_{is}u_{it}] = 0 \ \forall s \neq t$, and $E[\alpha_i u_{it}] = 0$.

Identification of the (structural) parameters λ, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ now crucially hinges on the assumptions about the dependencies between the regressors and the unit-specific effects.

Assumption 2: The explanatory variables can be decomposed as $\mathbf{x}_{it} = (\mathbf{x}'_{1it}, \mathbf{x}'_{2it})'$ and $\mathbf{f}_i = (\mathbf{f}'_{1i}, \mathbf{f}'_{2i})'$ such that $E[\alpha_i|\mathbf{x}_{1it}, \mathbf{f}_{1i}] = 0$, $E[\alpha_i|\mathbf{x}_{2it}] \neq 0$ and $E[\alpha_i|\mathbf{f}_{2i}] \neq 0$.

The resulting model is the dynamic counterpart of the Hausman and Taylor (1981) model. For further reference, the lengths of the subvectors are K_{x1}, K_{x2}, K_{f1}, and K_{f2}, respectively. If $K_{x2} = K_{f2} = 0$ the model collapses to the dynamic random effects model. Contrarily, $K_{x1} = 0$ and $K_{f1} = 1$ (the constant term) leads to the dynamic fixed effects model. In the remaining sections, we occasionally distinguish between strictly exogenous and predetermined regressors $\mathbf{x}_{it}$ with respect to the disturbance term u_{it}.

Assumption 3: The time-invariant regressors $\mathbf{f}_i$ are exogenous with respect to the disturbances u_{it}, while the time-varying regressors $\mathbf{x}_{it}$ can be strictly exogenous, $E[u_{it}|\mathbf{x}_{i0}, \mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT}, \mathbf{f}_i; \alpha_i] = 0$, or predetermined, $E[u_{it}|\mathbf{x}_{i0}, \mathbf{x}_{i1}, \ldots, \mathbf{x}_{it}, \mathbf{f}_i; \alpha_i] = 0$ and $E[u_{it}|\mathbf{x}_{is}] \neq 0 \ \forall s > t$.[3]

To facilitate the subsequent derivations we introduce the following notation. We can write model (1) as

$$\mathbf{y}_i = \lambda \mathbf{y}_{i,(-1)} + \mathbf{X}_i\boldsymbol{\beta} + \mathbf{F}_i\boldsymbol{\gamma} + \mathbf{e}_i, \quad \mathbf{e}_i = \alpha_i\boldsymbol{\iota}_T + \mathbf{u}_i, \tag{2}$$

where $\mathbf{y}_i = (y_{i1}, y_{i2}, \ldots, y_{iT})'$ is the vector of stacked observations of the dependent variable for unit i. $\mathbf{y}_{i,(-1)}, \mathbf{X}_i, \mathbf{F}_i$, $\mathbf{e}_i$, and $\mathbf{u}_i$ are defined accordingly. $\boldsymbol{\iota}_T$ is a $T \times 1$ vector of ones. When the

[3] For simplicity, we abstract from endogenous regressors with respect to u_{it}. They can be easily incorporated by adjusting the GMM moment conditions appropriately. See Blundell et al. (2000).

data is stacked for all units, for example $\mathbf{y} = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_N')'$, subscripts are omitted:

$$\mathbf{y} = \lambda \mathbf{y}_{(-1)} + \mathbf{X}\boldsymbol{\beta} + \mathbf{F}\boldsymbol{\gamma} + \mathbf{e}, \quad \mathbf{e} = \boldsymbol{\alpha} + \mathbf{u}. \tag{3}$$

Finally, let $\mathbf{W} = (\mathbf{y}_{(-1)}, \mathbf{X})$ be the matrix of time-varying regressors with corresponding coefficient vector $\boldsymbol{\theta} = (\lambda, \boldsymbol{\beta}')'$, and $\tilde{\mathbf{W}} = (\mathbf{y}_{(-1)}, \mathbf{X}, \mathbf{F})$ be the full regressor matrix.

3 One-Stage GMM Estimation

We can estimate all model parameters simultaneously by choosing appropriate instruments for the variables that are endogenous with respect to the unobserved unit-specific effects. In the following, we discuss generalized method of moments estimators that are based on the linear moment conditions

$$E[\mathbf{Z}_i'\mathbf{H}\mathbf{e}_i] = \mathbf{0}, \tag{4}$$

where $\mathbf{Z}_i$ is a matrix of K_z instruments, and $\mathbf{H}$ is a deterministic transformation matrix.

For the static model with strictly exogenous regressors $\mathbf{x}_{it}$, Hausman and Taylor (1981) propose an instrumental variable estimator that uses deviations from their within-group means, $\mathbf{x}_{it} - \bar{\mathbf{x}}_i$, as instruments for the regressors $\mathbf{x}_{it}$, and the within-group means $\bar{\mathbf{x}}_{1i}$ as instruments for $\mathbf{f}_{2i}$.[4] The time-invariant regressors $\mathbf{f}_{1i}$ serve as their own instruments. We can extend this estimator to the dynamic model by adding an appropriate instrument for the lagged dependent variable. For example, Anderson and Hsiao (1981) propose to use $y_{i,t-2}$ or $\Delta y_{i,t-2}$ as instruments for $\Delta y_{i,t-1}$. With $\mathbf{y}_{i,(-2)} = (y_{i0}, y_{i1}, \ldots, y_{i,T-2})'$, the resulting estimator satisfies the moment conditions (4) with

$$\mathbf{Z}_i = \begin{pmatrix} \mathbf{y}_{i,(-2)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_{1i} & \mathbf{F}_{1i} \end{pmatrix}, \quad \text{and} \quad \mathbf{H} = \begin{pmatrix} \mathbf{D} \\ \mathbf{Q} \\ \mathbf{P} \end{pmatrix},$$

for the $(T-1) \times T$ first-difference transformation matrix $\mathbf{D} = [(\mathbf{0}, \mathbf{I}_{T-1}) - (\mathbf{I}_{T-1}, \mathbf{0})]$, where $\mathbf{I}_{T-1}$ is the identity matrix of order $T-1$, and the $T \times T$ idempotent and symmetric projection matrices

[4] To improve on the efficiency of the estimator, Amemiya and MaCurdy (1986) propose to use all time periods of $\mathbf{x}_{1it}$ separately as instruments instead of the within-group means. Breusch et al. (1989) additionally suggest using the deviation of each individual time period from the within-group means as separate instruments.

$\mathbf{P} = \boldsymbol{\iota}_T(\boldsymbol{\iota}_T'\boldsymbol{\iota}_T)^{-1}\boldsymbol{\iota}_T'$ and $\mathbf{Q} = \mathbf{I}_T - \mathbf{P}$, where $\mathbf{P}$ and $\mathbf{Q}$ transform the observations into within-group means and deviations from within-group means, respectively. Importantly, both $\mathbf{D}$ and $\mathbf{Q}$ are orthogonal to time-invariant variables. Due to the block-diagonal structure of $\mathbf{Z}_i$, only the instruments $(\mathbf{X}_{1i}, \mathbf{F}_{1i})$ in the lower-right block of $\mathbf{Z}_i$ are of use to identify $\boldsymbol{\gamma}$. Therefore, as in the static model of Hausman and Taylor (1981), a necessary condition for the identification of all coefficients $(\boldsymbol{\theta}', \boldsymbol{\gamma}')'$ with this extended estimator is $K_{x1} \geq K_{f2}$.

Since the above estimator does not exploit all model implied moment conditions, it will be inefficient. Arellano and Bond (1991), Arellano and Bover (1995), and Blundell and Bond (1998) derive additional linear moment conditions for the model in first differences and in levels. Ahn and Schmidt (1995) add further moment conditions under homoscedasticity of u_{it} that are in part nonlinear. We present the full set of linear moment conditions in Appendix A. For the equations in first differences, $E[\mathbf{Z}_{di}'\mathbf{D}\mathbf{e}_i] = 0$, and in levels, $E[\mathbf{Z}_{li}'\mathbf{e}_i] = 0$, the moment conditions can be combined by defining

$$\mathbf{Z}_i = \begin{pmatrix} \mathbf{Z}_{di} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_{li} \end{pmatrix}, \quad \text{and} \quad \mathbf{H} = \begin{pmatrix} \mathbf{D} \\ \mathbf{I}_T \end{pmatrix}$$

in equation (4). It is well documented by Blundell and Bond (1998) and others (in the absence of time-invariant regressors) that the GMM estimator with instruments for the first-differenced equation only suffers from a potentially severe weak instruments problem when $\lambda \to 1$. Under an additional mean stationarity assumption, Assumption 5 in Appendix A, they suggest to additionally use the first differences of the time-varying variables as instruments for the equation in levels. However, Bun and Windmeijer (2010) demonstrate that these instruments also can become weak, in particular when the variance ratio of the unit-specific effects relative to the idiosyncratic error term exceeds unity. To the contrary, the instruments for the first-differenced equation may regain strength when mean stationarity is not satisfied, as demonstrated by Hayakawa (2009).

Yet, since $\mathbf{D}\boldsymbol{\iota}_T = \mathbf{0}$, the instruments that are relevant for the identification of the coefficients $\boldsymbol{\gamma}$ need to be placed in $\mathbf{Z}_{li}$. Following Arellano and Bond (1991) and Arellano and Bover (1995), the following $K_{x1}(T+1) + K_{f1}$ non-redundant linear moment conditions arise under Assumption

2 for the model in levels:

$$E[\mathbf{x}_{1i0}e_{i1}] = \mathbf{0}, \quad \text{and} \quad E[\mathbf{x}_{1it}e_{it}] = \mathbf{0}, \quad t = 1, 2, \ldots, T, \tag{5}$$

$$E\left[\sum_{t=1}^{T} \mathbf{f}_{1i}e_{it}\right] = \mathbf{0}. \tag{6}$$

Consequently, in the absence of external instruments a necessary condition for the identification of all coefficients $(\boldsymbol{\theta}', \boldsymbol{\gamma}')'$ in equation (1) is that $K_{x1}(T+1) \geq K_{f2}$.[5] Because levels instead of first differences of the variables $\mathbf{x}_{1it}$ (and $\mathbf{f}_{1i}$) are used in the moment conditions (5) and (6), the aforementioned weak instruments problem by Bun and Windmeijer (2010) is not an issue here. Nevertheless, a general weak correlation problem of the instruments $\mathbf{x}_{1it}$ with the instrumented regressors $\mathbf{f}_{2i}$ might still occur.

Remark 1: In practice, it will often be hard to justify that separate time periods of the exogenous time-varying regressors provide sufficient explanatory power for the instrumented time-invariant regressors after partialling out the initial observations or within-group means, that is $E[\mathbf{f}_{2i}|\mathbf{x}_{1i0}, \mathbf{X}_{1i}, \mathbf{f}_{1i}] = E[\mathbf{f}_{2i}|\mathbf{x}_{1i0}, \mathbf{f}_{1i}]$ or $E[\mathbf{f}_{2i}|\mathbf{x}_{1i0}, \mathbf{X}_{1i}, \mathbf{f}_{1i}] = E[\mathbf{f}_{2i}|\bar{\mathbf{x}}_{1i}, \mathbf{f}_{1i}]$. The identification condition then tightens again to $K_{x1} \geq K_{f2}$.

Define $\tilde{\mathbf{H}} = \mathbf{I}_N \otimes \mathbf{H}$, where $\otimes$ denotes the Kronecker product. Based on the sample moments $N^{-1}\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{e}$, we can now derive the GMM estimator that minimizes the following distance function:

$$\begin{pmatrix} \hat{\boldsymbol{\theta}} \\ \hat{\gamma} \end{pmatrix} = \arg\min_{\boldsymbol{\theta}, \boldsymbol{\gamma}} \mathbf{e}'\tilde{\mathbf{H}}'\mathbf{Z}\mathbf{V}_N\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{e},$$

where $\mathbf{V}_N$ is a positive definite weighting matrix. If all elements in $(\boldsymbol{\theta}', \boldsymbol{\gamma}')'$ are identified, that is $\tilde{\mathbf{W}}'\tilde{\mathbf{H}}'\mathbf{Z}\mathbf{V}_N\mathbf{Z}'\tilde{\mathbf{H}}\tilde{\mathbf{W}}$ is non-singular, we obtain

$$\begin{pmatrix} \hat{\boldsymbol{\theta}} \\ \hat{\gamma} \end{pmatrix} = \left(\tilde{\mathbf{W}}'\tilde{\mathbf{H}}'\mathbf{Z}\mathbf{V}_N\mathbf{Z}'\tilde{\mathbf{H}}\tilde{\mathbf{W}}\right)^{-1} \tilde{\mathbf{W}}'\tilde{\mathbf{H}}'\mathbf{Z}\mathbf{V}_N\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{y}. \tag{7}$$

[5] External instruments can be incorporated in a straightforward way.

The following familiar result under the data generating process (1) applies:[6]

Lemma 1: If the moment conditions (4) are satisfied and all coefficients are identified, then under standard regularity conditions the joint asymptotic distribution of the one-stage GMM estimator (7) is:

$$\sqrt{N}\begin{pmatrix}\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\\ \hat{\boldsymbol{\gamma}}-\boldsymbol{\gamma}\end{pmatrix}\overset{a}{\sim}\mathcal{N}(\mathbf{0},\Sigma), \tag{8}$$

with

$$\Sigma=(\mathbf{S}'\mathbf{V}\mathbf{S})^{-1}\mathbf{S}'\mathbf{V}\Xi\mathbf{V}\mathbf{S}(\mathbf{S}'\mathbf{V}\mathbf{S})^{-1}, \tag{9}$$

where $\mathbf{S}=\operatorname{plim}N^{-1}\mathbf{Z}'\tilde{\mathbf{H}}\tilde{\mathbf{W}}$, $\Xi=\operatorname{plim}N^{-1}\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{e}\mathbf{e}'\tilde{\mathbf{H}}'\mathbf{Z}$, and $\mathbf{V}=\operatorname{plim}\mathbf{V}_N$.

From equation (9) in Lemma 1 we can infer the following statement on the efficiency of the GMM estimator:[7]

Lemma 2: The GMM estimator is asymptotically efficient for a given instruments matrix $\mathbf{Z}$ and transformation matrix $\tilde{\mathbf{H}}$ if $\mathbf{V}=\Xi^{-1}$.

Blundell and Bond (1998) and Windmeijer (2000) emphasize that for dynamic panel data models, in general, efficient GMM estimation is infeasible without having a prior estimate of Ξ. A feasible efficient GMM estimator can be obtained in two steps. In the first step, choosing any positive definite matrix $\mathbf{V}_N$ will yield consistent but generally inefficient estimates $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\gamma}}$. The second-step estimator is then based on $\mathbf{V}_N=\hat{\Xi}^{-1}$. A consistent unrestricted estimate of Ξ is obtained as $\hat{\Xi}=N^{-1}\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{H}\hat{\mathbf{e}}_i\hat{\mathbf{e}}_i'\mathbf{H}'\mathbf{Z}_i$, with $\hat{\mathbf{e}}_i=\mathbf{y}_i-\mathbf{W}_i\hat{\boldsymbol{\theta}}-\mathbf{F}_i\hat{\boldsymbol{\gamma}}$.[8] The importance of choosing an appropriate first-step weighting matrix should not be underestimated in applied work. Although the second-step GMM estimator is asymptotically unaffected, its finite sample performance still depends on the choice of $\mathbf{V}_N$ in the first step. Windmeijer (2005) shows that asymptotic standard error estimates of the two-step GMM estimator can be severely downward biased in finite samples. He derives a finite sample variance correction. Alternatives to the two-step GMM estimator that

[6] See for instance Hansen (1982), Theorem 3.1, or Newey and McFadden (1994), Theorem 3.4.

[7] This result dates back to Hansen (1982), Theorem 3.2, and was generalized by Newey and McFadden (1994), Theorem 5.2.

[8] For more details on efficient GMM estimation see Appendix B.

are targeted to improve the finite sample performance include the iterated and the continuously updated GMM estimators, see for example Hansen et al. (1996).

Moreover, GMM estimators might suffer from severe finite sample distortions that arise from having too many instruments relative to the sample size, as stressed by Roodman (2009) among others. The instrument count can be reduced by forming linear combinations $\mathbf{Z}_i\mathbf{R}$ of the columns of $\mathbf{Z}_i$. For any deterministic transformation matrix $\mathbf{R}$, this also leads to a valid set of moment conditions, $E[\mathbf{R}'\mathbf{Z}_i'\mathbf{H}\mathbf{e}_i] = \mathbf{0}$. The GMM estimator (7) is then based on the transformed instruments $\mathbf{Z}_i\mathbf{R}$. We provide examples of relevant transformation matrices in Appendix C.

4 Two-Stage Estimation

When estimating all regression coefficients simultaneously, a misclassification of time-invariant regressors as being uncorrelated with the unit-specific effects might lead to a biased estimation of all coefficients including λ and β. In this section, we lay down a robust two-stage estimation procedure. In a first stage, we subsume the time-invariant variables $\mathbf{f}_i$ under the unit-specific effects, $\tilde{\alpha}_i = \alpha_i + \mathbf{f}_i'\boldsymbol{\gamma}$, and consistently estimate the coefficients λ and $\boldsymbol{\beta}$ independent of the assumptions on the correlation structure between $\mathbf{f}_i$ and α_i. In the second stage, we recover γ.

The first-stage model is

$$y_{it} = \lambda y_{i,t-1} + \mathbf{x}_{it}'\boldsymbol{\beta} + \bar{\alpha} + \tilde{e}_{it}, \quad \tilde{e}_{it} = \tilde{\alpha}_i - \bar{\alpha} + u_{it}, \tag{10}$$

where $\bar{\alpha} = E[\tilde{\alpha}_i]$. To obtain the first-stage estimates $\hat{\lambda}$ and $\hat{\boldsymbol{\beta}}$ we can apply a transformation that eliminates the time-invariant unit-specific effects $\tilde{\alpha}_i$. In particular, the GMM estimator of Arellano and Bond (1991) and the QML estimator of Hsiao et al. (2002) are based on the first-differenced model, while Arellano and Bover (1995) propose a GMM estimator based on forward orthogonal deviations. Alternatively, system GMM estimators as discussed in Section 3 that also make use of the level relationship can be applied taking into account that the time-invariant variables $\mathbf{f}_i$ are now part of the first-stage error term $\tilde{e}_{it}$. If $K_{x1} > 0$ but some or all of the variables in $\mathbf{x}_{1it}$ are correlated with $\mathbf{f}_i$ then these variables are uncorrelated with α_i but not with $\tilde{\alpha}_i$. Hence, the first-stage instruments need to be adjusted appropriately. We do not restrict the analysis to any particular first-stage estimator but make the following assumption:[9]

Assumption 4: $\hat{\boldsymbol{\theta}}$ is a consistent asymptotically linear first-stage estimator with influence function $\boldsymbol{\psi}_i$ such that

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \boldsymbol{\psi}_i + o_p(1), \tag{11}$$

$E[\boldsymbol{\psi}_i] = 0$, and $E[\boldsymbol{\psi}_i \boldsymbol{\psi}_i'] = \Sigma_\theta$.

Asymptotic normality of $\hat{\boldsymbol{\theta}}$ follows under standard regularity conditions.[10] Also, denote $\boldsymbol{\psi} = \sum_{i=1}^{N} \boldsymbol{\psi}_i$.

In the second stage, we estimate the coefficients $\boldsymbol{\gamma}$ of the time-invariant variables based on the level relationship:

$$y_{it} - \hat{\lambda} y_{i,t-1} - \mathbf{x}_{it}' \hat{\boldsymbol{\beta}} = \mathbf{f}_i' \boldsymbol{\gamma} + v_{it}, \quad v_{it} = \alpha_i + u_{it} - (\hat{\lambda} - \lambda) y_{i,t-1} - \mathbf{x}_{it}' (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}). \tag{12}$$

In particular, note the two additional terms in the error term v_{it} that are due to the first-stage estimation error such that this second-stage error term is no longer independent and identically distributed. We can now set up a second-stage GMM estimator based on the asymptotic moment conditions

$$\lim_{N \to \infty} E \left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{Z}_{\gamma i}' \mathbf{v}_i \right] = \mathbf{0}. \tag{13}$$

Under Assumption 2, we can use the observations $\mathbf{x}_{1it}$ as instruments for the endogenous regressors $\mathbf{f}_{2i}$. The resulting non-redundant asymptotic moment conditions are similar to those given by equations (5) and (6):

$$\lim_{N \to \infty} E \left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{1i0} v_{i1} \right] = \mathbf{0}, \quad \text{and} \quad \lim_{N \to \infty} E \left[\frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{1it} v_{it} \right] = \mathbf{0}, \quad t = 1, 2, \ldots, T, \tag{14}$$

$$\lim_{N \to \infty} E \left[\frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \mathbf{f}_{1i} v_{it} \right] = \mathbf{0}. \tag{15}$$

[9] We pick up the case of a first-stage GMM estimator in the next section. Two-stage QML estimation is briefly discussed in Appendix E.

[10] Compare Newey and McFadden (1994), Chapter 3.

The corresponding instruments matrix is given as $\mathbf{Z}_{\gamma i} = (\mathbf{Z}_{xi}, \mathbf{F}_{1i})$, with

$$\mathbf{Z}_{xi} = \begin{pmatrix} \mathbf{x}'_{1i0} & \mathbf{x}'_{1i1} & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{x}'_{1i2} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{x}'_{1iT} \end{pmatrix},$$

which is valid both for strictly exogenous and predetermined variables $\mathbf{x}_{1it}$. Consequently, the order condition from the previous section transmits to the second-stage GMM estimation: A necessary condition for the identification of the coefficients $\boldsymbol{\gamma}$ in equation (12) is that $K_{x1}(T+1) \geq K_{f2}$.[11] The second-stage GMM estimator then solves[12]

$$\hat{\hat{\boldsymbol{\gamma}}} = \arg\min_{\boldsymbol{\gamma}} \mathbf{v}'\mathbf{Z}_\gamma \mathbf{V}_{\gamma N} \mathbf{Z}'_\gamma \mathbf{v},$$

for a positive definite weighting matrix $\mathbf{V}_{\gamma N}$. When $\boldsymbol{\gamma}$ is identified, the second-stage GMM estimator is given by:

$$\hat{\hat{\boldsymbol{\gamma}}} = \left(\mathbf{F}'\mathbf{Z}_\gamma \mathbf{V}_{\gamma N} \mathbf{Z}'_\gamma \mathbf{F}\right)^{-1} \mathbf{F}'\mathbf{Z}_\gamma \mathbf{V}_{\gamma N} \mathbf{Z}'_\gamma (\mathbf{y} - \mathbf{W}\hat{\boldsymbol{\theta}}). \tag{16}$$

We can now formulate the following proposition:

Proposition 1: If Assumption 4 holds, the moment conditions (4) are satisfied and all coefficients are identified, then under standard regularity conditions the asymptotic distribution of the second-stage GMM estimator (16) is:

$$\sqrt{N}\left(\hat{\hat{\boldsymbol{\gamma}}} - \boldsymbol{\gamma}\right) \overset{a}{\sim} \mathcal{N}\left(\mathbf{0}, \Sigma_\gamma\right), \tag{17}$$

with

$$\Sigma_\gamma = (\mathbf{S}'_F \mathbf{V}_\gamma \mathbf{S}_F)^{-1} \mathbf{S}_F{}' \mathbf{V}_\gamma \Xi_v \mathbf{V}_\gamma \mathbf{S}_F (\mathbf{S}'_F \mathbf{V}_\gamma \mathbf{S}_F)^{-1}, \tag{18}$$

where $\mathbf{S}_F = \operatorname{plim} N^{-1}\mathbf{Z}'_\gamma \mathbf{F}$, $\Xi_v = \operatorname{plim} N^{-1}\mathbf{Z}'_\gamma \mathbf{v}\mathbf{v}'\mathbf{Z}_\gamma$, and $\mathbf{V}_\gamma = \operatorname{plim} \mathbf{V}_{\gamma N}$. Moreover,

$$\Xi_v = \Xi_e + \mathbf{S}_W \Sigma_\theta \mathbf{S}'_W - \Xi'_{\theta e}\mathbf{S}'_W - \mathbf{S}_W \Xi_{\theta e}, \tag{19}$$

[11] The qualifications of Remark 1 apply again.

[12] A double hat denotes second-stage estimates while a single hat refers to first-stage estimates.

where $\mathbf{S}_W = \text{plim}\, N^{-1}\mathbf{Z}_\gamma'\mathbf{W}$, $\Xi_e = \text{plim}\, N^{-1}\mathbf{Z}_\gamma'\mathbf{e}\mathbf{e}'\mathbf{Z}_\gamma$, and $\Xi_{\theta e} = \text{plim}\, N^{-1}\boldsymbol{\psi}\mathbf{e}'\mathbf{Z}_\gamma$.

Proof. Inserting model (3) into equation (16) and scaling by $\sqrt{N}$ we obtain:

$$\begin{aligned}\sqrt{N}\left(\hat{\hat{\boldsymbol{\gamma}}} - \boldsymbol{\gamma}\right) &= \left[\left(\frac{1}{N}\mathbf{F}'\mathbf{Z}_\gamma\right)\mathbf{V}_{\gamma N}\left(\frac{1}{N}\mathbf{Z}_\gamma'\mathbf{F}\right)\right]^{-1}\left(\frac{1}{N}\mathbf{F}'\mathbf{Z}_\gamma\right)\mathbf{V}_{\gamma N}\left(\frac{1}{\sqrt{N}}\mathbf{Z}_\gamma'\mathbf{v}\right) \\ &= (\mathbf{S}_F'\mathbf{V}_\gamma\mathbf{S}_F)^{-1}\mathbf{S}_F{}'\mathbf{V}_\gamma\left[\frac{1}{\sqrt{N}}\mathbf{Z}_\gamma'\mathbf{e} - \mathbf{S}_W\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})\right] + o_p(1) \\ &= (\mathbf{S}_F'\mathbf{V}_\gamma\mathbf{S}_F)^{-1}\mathbf{S}_F{}'\mathbf{V}_\gamma\left[\frac{1}{\sqrt{N}}\sum_{i=1}^N(\mathbf{Z}_{\gamma i}'\mathbf{e}_i - \mathbf{S}_W\boldsymbol{\psi}_i)\right] + o_p(1),\end{aligned}$$

where the last equality follows from Assumption 4. By applying the central limit theorem, $N^{-1/2}\sum_{i=1}^N(\mathbf{Z}_{\gamma i}'\mathbf{e}_i - \mathbf{S}_W\boldsymbol{\psi}_i) \overset{a}{\sim} \mathcal{N}(\mathbf{0}, \Xi_e + \mathbf{S}_W\Sigma_\theta\mathbf{S}_W' - \Xi_{\theta e}'\mathbf{S}_W' - \mathbf{S}_W\Xi_{\theta e})$, and equation (18) follows from the continuous mapping theorem.[13] □

Remark 2: For completeness, the asymptotic covariance matrix between the first-stage and the second-stage estimator is given by

$$\lim_{N\to\infty} E\left[\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)\left(\hat{\hat{\boldsymbol{\gamma}}} - \boldsymbol{\gamma}\right)'\right] = (\Sigma_\theta\mathbf{S}_W' + \Xi_{\theta e})\mathbf{V}_\gamma\mathbf{S}_F(\mathbf{S}_F'\mathbf{V}_\gamma\mathbf{S}_F)^{-1}. \tag{20}$$

In analogy to Lemma 2, we can state the following corollary:

Corollary 1: The second-stage GMM estimator $\hat{\hat{\boldsymbol{\gamma}}}$ is efficient for a given first-stage estimator $\hat{\boldsymbol{\theta}}$ and instruments matrix $\mathbf{Z}_\gamma$ if $\mathbf{V}_\gamma = \Xi_v^{-1}$.

Similar to one-stage GMM estimators, feasible efficient estimation requires an initial estimate of Ξ_v unless $\mathbf{Z}_\gamma'\mathbf{F}$ is non-singular. A consistent unrestricted estimate of Ξ is obtained as

$$\hat{\hat{\Xi}}_v = \hat{\hat{\Xi}}_e + \hat{\hat{\mathbf{S}}}_W\hat{\Sigma}_\theta\hat{\hat{\mathbf{S}}}_W' - \hat{\hat{\Xi}}_{\theta e}'\hat{\hat{\mathbf{S}}}_W' - \hat{\hat{\mathbf{S}}}_W\hat{\hat{\Xi}}_{\theta e}, \tag{21}$$

where $\hat{\hat{\mathbf{S}}}_W = N^{-1}\mathbf{Z}_\gamma'\mathbf{W}$. An estimate of Σ_θ is readily available from the first-stage regression. An estimate of Ξ_e can be obtained as $\hat{\hat{\Xi}}_e = N^{-1}\sum_{i=1}^N \mathbf{Z}_{\gamma i}'\hat{\hat{\mathbf{e}}}_i\hat{\hat{\mathbf{e}}}_i'\mathbf{Z}_{\gamma i}$, where $\hat{\hat{\mathbf{e}}}_i = \mathbf{y}_i - \mathbf{W}_i\hat{\boldsymbol{\theta}} - \mathbf{F}_i\hat{\hat{\boldsymbol{\gamma}}}$ for a consistent initial estimate $\hat{\hat{\boldsymbol{\gamma}}}$. Obtaining an estimate of $\Xi_{\theta e}$ is more involved as it relies on the

[13] Compare Newey and McFadden (1994), Chapter 6.

product of the influence function $\boldsymbol{\psi}_i$ from the first stage and the moment function from the second stage:[14]

$$\hat{\hat{\Xi}}_{\theta e} = \frac{1}{N} \sum_{i=1}^{N} \hat{\boldsymbol{\psi}}_i \hat{\hat{\mathbf{e}}}_i' \mathbf{Z}_{\gamma i}. \tag{22}$$

Importantly, ignoring the first-stage estimation error by setting $\hat{\hat{\Xi}}_v = \hat{\hat{\Xi}}_e$ might not only yield an inefficient second-stage estimator but also produces inconsistent standard error estimates of $\hat{\hat{\boldsymbol{\gamma}}}$. In general, the direction of the bias of uncorrected standard errors is a priori unclear unless $\Xi_{\theta e} = \mathbf{0}$. In the latter case, the difference $\Xi_v - \Xi_e = \mathbf{S}_W \Sigma_\theta \mathbf{S}_W'$ is a positive semi-definite matrix and, consequently, standard error estimates ignoring the correction term will be too small.[15] $\Xi_{\theta e} = \mathbf{0}$ holds for example in the special case where we consider a first-stage GMM estimator that uses moment conditions for the first-differenced model only, that is $\mathbf{H} = \mathbf{D}$, all second-stage instruments $\mathbf{Z}_{\gamma i}$ are time-invariant, and the errors u_{it} are independent and homoscedastic across units and time. Finally, ignoring the first stage is only valid if $\mathbf{S}_W = \mathbf{0}$.

Remark 3: As an alternative to the strong Assumption 2 that requires some regressors to be uncorrelated with the unobserved unit-specific effects α_i in model (1), we can consider a correlated random effects assumption in the spirit of Mundlak (1978), $E[\alpha_i|\mathbf{X}_i, \mathbf{F}_i] = b + \bar{\mathbf{x}}_i'\boldsymbol{\pi}$, or Chamberlain (1982), $E[\alpha_i|\mathbf{X}_i, \mathbf{F}_i] = b + \sum_{s=0}^{T} \mathbf{x}_{is}'\boldsymbol{\pi}_s$. Notice that the time-invariant regressors are part of the conditioning set but do not appear at the right-hand side. Either of these assumptions allows the time-varying regressors to be correlated with the unobserved effects, although not in an arbitrary way. The time-invariant regressors are as well allowed to be correlated with them but only indirectly through their correlation with the within-group means $\bar{\mathbf{x}}_i$ or some linear combination of the observations $\mathbf{x}_{is}$. Taking for example the Mundlak (1978) approach, we can then replace $\alpha_i = b + \bar{\mathbf{x}}_i'\boldsymbol{\pi} + \eta_i$ in the second-stage equation (12) and treat $\bar{\mathbf{x}}_i$ as additional time-invariant regressors. In this situation, all time-invariant variables serve as instruments for themselves and the coefficients $\boldsymbol{\gamma}$ can be consistently estimated at the second stage (besides the regression constant for which we obtain $\text{plim}\, \hat{\hat{\gamma}}_1 = \gamma_1 + b$).

[14] We derive the influence function for a first-stage GMM estimator in Appendix D and for a first-stage QML estimator in Appendix E.

[15] A generalization of this result can be found in Newey (1984).

5 One-Stage versus Two-Stage GMM Estimation

We are now in a position to shed more light on one-stage and two-stage GMM estimators and to contrast the two. To facilitate the following exposition, denote by $(\hat{\boldsymbol{\theta}}_s', \hat{\gamma}_s')'$ the one-stage system GMM estimator (7) and decompose its weighting matrix $\mathbf{V}_N = \mathbf{L}\mathbf{L}'$ with $\text{rk}(\mathbf{L}) = K_z$. Also define $\mathbf{y}^* = \mathbf{L}'\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{y}$, $\mathbf{W}^* = \mathbf{L}'\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{W}$, and $\mathbf{F}^* = \mathbf{L}'\mathbf{Z}'\tilde{\mathbf{H}}\mathbf{F}$. The following partitioned regression result will be helpful:

$$\hat{\boldsymbol{\theta}}_s = (\mathbf{W}^{*\prime}\mathbf{M}_F\mathbf{W}^*)^{-1}\mathbf{W}^{*\prime}\mathbf{M}_F\mathbf{y}^*, \tag{23}$$

$$\hat{\gamma}_s = (\mathbf{F}^{*\prime}\mathbf{F}^*)^{-1}\mathbf{F}^{*\prime}\left(\mathbf{y}^* - \mathbf{W}^*\hat{\boldsymbol{\theta}}\right), \tag{24}$$

where $\mathbf{M}_F = \mathbf{I}_{K_z} - \mathbf{F}^*(\mathbf{F}^{*\prime}\mathbf{F}^*)^{-1}\mathbf{F}^{*\prime}$ is an idempotent and symmetric projection matrix. Furthermore, partition the weighting matrix as

$$\mathbf{V}_N = \begin{pmatrix} \mathbf{V}_{dN} & \mathbf{V}_{dlN} \\ \mathbf{V}_{dlN}' & \mathbf{V}_{lN} \end{pmatrix}, \tag{25}$$

conformable for multiplications $\mathbf{Z}_d\mathbf{V}_{dN}\mathbf{Z}_d'$ and $\mathbf{Z}_l\mathbf{V}_{lN}\mathbf{Z}_l'$. As an alternative consider the two-stage GMM estimator $(\hat{\boldsymbol{\theta}}_d', \hat{\hat{\gamma}}_d')'$, where $\hat{\boldsymbol{\theta}}_d$ is based on the moment conditions $E[\mathbf{Z}_{di}'\mathbf{D}\mathbf{e}_i] = \mathbf{0}$ for the transformed model only, and with weighting matrix $\mathbf{V}_{\theta N}$:

$$\hat{\boldsymbol{\theta}}_d = \left(\mathbf{W}'\tilde{\mathbf{D}}'\mathbf{Z}_d\mathbf{V}_{\theta N}\mathbf{Z}_d'\tilde{\mathbf{D}}\mathbf{W}\right)^{-1}\mathbf{W}'\tilde{\mathbf{D}}'\mathbf{Z}_d\mathbf{V}_{\theta N}\mathbf{Z}_d'\tilde{\mathbf{D}}\mathbf{y}, \tag{26}$$

where $\tilde{\mathbf{D}} = \mathbf{I}_N \otimes \mathbf{D}$. The second-stage estimator $\hat{\hat{\gamma}}_d$ is given by equation (16) based on $\hat{\boldsymbol{\theta}}_d$ in the first stage. We can now make the following claim:

Proposition 2: It holds that $\hat{\boldsymbol{\theta}}_s = \hat{\boldsymbol{\theta}}_d$, with $\hat{\boldsymbol{\theta}}_s$ and $\hat{\boldsymbol{\theta}}_d$ given by equations (23) and (26), respectively, if $\mathbf{Z}_l'\mathbf{F}$ is non-singular and $\mathbf{V}_{\theta N} = \mathbf{V}_{dN} - \mathbf{V}_{dlN}\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'$.

Proof. Observe that $\mathbf{F}'\tilde{\mathbf{H}}'\mathbf{Z} = (\mathbf{F}'\tilde{\mathbf{D}}'\mathbf{Z}_d, \mathbf{F}'\mathbf{Z}_l) = (\mathbf{0}, \mathbf{F}'\mathbf{Z}_l)$ since $\tilde{\mathbf{D}}\mathbf{F} = \mathbf{0}$. Consequently, $\mathbf{F}^{*\prime}\mathbf{F}^* = \mathbf{F}'\mathbf{Z}_l\mathbf{V}_{lN}\mathbf{Z}_l'\mathbf{F}$. With $\mathbf{Z}_l'\mathbf{F}$ being non-singular, it follows that $(\mathbf{F}^{*\prime}\mathbf{F}^*)^{-1} = (\mathbf{Z}_l'\mathbf{F})^{-1}\mathbf{V}_{lN}^{-1}(\mathbf{F}'\mathbf{Z}_l)^{-1}$.

Let $\mathbf{V}_{\theta N} = \mathbf{V}_{dN} - \mathbf{V}_{dlN}\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'$. Then,

$$\mathbf{L}\mathbf{M}_F\mathbf{L}' = \mathbf{V}_N - \mathbf{V}_N \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{lN}^{-1} \end{pmatrix} \mathbf{V}_N = \begin{pmatrix} \mathbf{V}_{\theta N} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix},$$

such that after straightforward algebra equation (23) boils down to equation (26). Alternatively, if $\mathbf{Z}_d'\tilde{\mathbf{D}}'\mathbf{W}$ is non-singular as well, $\hat{\boldsymbol{\theta}}_s = \hat{\boldsymbol{\theta}}_d = (\mathbf{Z}_d'\tilde{\mathbf{D}}'\mathbf{W})^{-1}\mathbf{Z}_d'\tilde{\mathbf{D}}'\mathbf{y}$ independent of the choice of the weighting matrices. □

When $\mathbf{Z}_l'\mathbf{F}$ is non-singular, the coefficients $\boldsymbol{\gamma}$ are exactly identified because the time-invariant regressors are orthogonal to the instruments for the first-differenced equation. But then the instruments for the level equation cannot be used any more to identify the coefficients $\boldsymbol{\theta}$, and $\hat{\boldsymbol{\theta}}_s$ consequently equals $\hat{\boldsymbol{\theta}}_d$ with an appropriate choice of the weighting matrix. A similar proposition holds for the coefficients $\boldsymbol{\gamma}$ under the additional restriction that the level instruments of the one-stage system GMM estimator equal the instruments of the second-stage GMM estimator, $\mathbf{Z}_l = \mathbf{Z}_\gamma$:

Proposition 3: With $\mathbf{Z}_l = \mathbf{Z}_\gamma$, it holds that $\hat{\gamma}_s = \hat{\hat{\gamma}}_d$, with $\hat{\gamma}_s$ and $\hat{\hat{\gamma}}_d$ given by equations (24) and (16), respectively, if $\mathbf{Z}_\gamma'\mathbf{F}$ is non-singular, $\mathbf{V}_{\theta N} = \mathbf{V}_{dN}$, and $\mathbf{V}_{dlN} = \mathbf{0}$.

Proof. With $\mathbf{F}^{*\prime}\mathbf{F}^* = \mathbf{F}'\mathbf{Z}_l\mathbf{V}_{lN}\mathbf{Z}_l'\mathbf{F}$ and $\mathbf{Z}_l = \mathbf{Z}_\gamma$, equation (24) can be written as

$$\hat{\gamma}_s = (\mathbf{F}'\mathbf{Z}_\gamma\mathbf{V}_{lN}\mathbf{Z}_\gamma'\mathbf{F})^{-1}\mathbf{F}'\mathbf{Z}_\gamma\mathbf{V}_{lN}(\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'\mathbf{Z}_d'\tilde{\mathbf{D}} + \mathbf{Z}_\gamma')(\mathbf{y} - \mathbf{W}\hat{\boldsymbol{\theta}}_s).$$

With $\mathbf{Z}_\gamma'\mathbf{F}$ being non-singular, this equation reduces further to

$$\hat{\gamma}_s = (\mathbf{Z}_\gamma'\mathbf{F})^{-1}(\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'\mathbf{Z}_d'\tilde{\mathbf{D}} + \mathbf{Z}_\gamma')(\mathbf{y} - \mathbf{W}\hat{\boldsymbol{\theta}}_s).$$

Also, equation (16) becomes $\hat{\hat{\gamma}}_d = (\mathbf{Z}_\gamma'\mathbf{F})^{-1}\mathbf{Z}_\gamma'(\mathbf{y} - \mathbf{W}\hat{\boldsymbol{\theta}}_d)$ independent of $\mathbf{V}_{\gamma N}$. Consequently, $\hat{\gamma}_s = \hat{\hat{\gamma}}_d$ if $\mathbf{V}_{dlN} = \mathbf{0}$ and $\hat{\boldsymbol{\theta}}_s = \hat{\boldsymbol{\theta}}_d$. The latter results as a consequence of Proposition 2 by setting $\mathbf{V}_{\theta N} = \mathbf{V}_{dN} - \mathbf{V}_{dlN}\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}' = \mathbf{V}_{dN}$. Alternatively, if $\mathbf{Z}_d'\tilde{\mathbf{D}}'\mathbf{W}$ is non-singular as well, $\mathbf{Z}_d'\tilde{\mathbf{D}}(\mathbf{y} - \mathbf{W}\hat{\boldsymbol{\theta}}_d) = \mathbf{0}$ and again $\hat{\boldsymbol{\theta}}_s = \hat{\boldsymbol{\theta}}_d$ without any restriction on the weighting matrices. □

Taken together, Propositions 2 and 3 state that one-stage and two-stage GMM estimation are equivalent for a particular choice of the weighting matrices if both utilize the same linearly independent instruments for the equation in levels and their number equals the count of time-invariant regressors. In this case, the first-stage GMM estimator of the two-stage approach is based on the moment conditions for the transformed model only. Leaving aside the trivial case of exact identification of the coefficients $\boldsymbol{\theta}$ as well, we can now infer a statement on asymptotic efficiency. When $\mathbf{V}_N$ is the optimal weighting matrix for the estimator $\hat{\boldsymbol{\theta}}_s$ according to Lemma 2, then an optimal weighting matrix for the estimator $\hat{\boldsymbol{\theta}}_d$ is given by $\mathbf{V}_{\theta N} = \mathbf{V}_{dN} - \mathbf{V}_{dlN}\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'$ as can be easily seen by calculating the partitioned inverse of $\mathbf{V}_N$. This corresponds to the condition that is required by Proposition 2. However, for equivalence of the one-stage and the two-stage estimators, Proposition 3 requires a block-diagonal weighting matrix $\mathbf{V}_N$ of the one-stage estimator such that $\mathbf{V}_{dlN} = \mathbf{0}$. It is clear that this restricted estimator is less efficient than the feasible efficient one-stage GMM estimator in general unless the optimal one-stage weighting matrix is indeed block-diagonal asymptotically. A relevant case where this holds is a restricted covariance structure of the error term, $E[\mathbf{e}_i\mathbf{e}_i'|\mathbf{Z}_i] = \sigma_\alpha^2 \boldsymbol{\iota}_T\boldsymbol{\iota}_T' + \sigma_u^2\mathbf{I}_T$, together with time-invariance of the level instruments $\mathbf{Z}_{li}$. In this case, the feasible efficient one-stage and two-stage GMM estimators will be (asymptotically) identical, and therefore also have the same variance.

Remark 4: If the optimal weighting matrices $\mathbf{V}_N$ or $\mathbf{V}_{\theta N}$ are based on separate initial consistent estimates (of σ_u^2), the equivalence of $\mathbf{V}_{\theta N}$ and $\mathbf{V}_{dN} - \mathbf{V}_{dlN}\mathbf{V}_{lN}^{-1}\mathbf{V}_{dlN}'$ only holds asymptotically, and the resulting feasible efficient estimators can be numerically different in finite samples, even if all other conditions of Propositions 2 and 3 are satisfied.

If the moment conditions for the level equation outnumber the time-invariant regressors, the one-stage and the two-stage GMM estimators will generally be different because the information contained in the level instruments $\mathbf{Z}_{li}$ is no longer exclusively used to identify $\boldsymbol{\gamma}$. A clear ranking of the two estimators in terms of efficiency is not possible anymore. Also, a misspecification of the level moment conditions might now turn the coefficient estimates for the time-varying regressors inconsistent.

6 Testing the Overidentifying Restrictions

For the identification of the coefficients of the time-invariant regressors, Assumption 2 is crucial, and a testing procedure for the validity of the regressor classification is desirable. Whenever the model parameters are overidentified, we can proceed along the lines of the Hansen (1982) test. If we cannot reject the null hypothesis of joint validity of the overidentifying restrictions for the one-stage GMM estimator (7), this suggests that the model is correctly specified. A rejection of the test, however, is not very informative about the source of misspecification due to the typically large number of overidentifying restrictions. Besides a wrong classification of regressors as being uncorrelated with the unit-specific effects under the Hausman and Taylor (1981) Assumption 2, other reasons might be undetected serial correlation of the errors,[16] a misclassification of predetermined (or endogenous) time-varying regressors as strictly exogenous, or a violation of the mean stationarity Assumption 5.

With regard to the moment conditions (5) and (6) that are of particular interest, a difference-in-Hansen test for a subset of the moment conditions as proposed by Eichenbaum et al. (1988) is not helpful either. The coefficients $\boldsymbol{\gamma}$ will be generally unidentified under the restricted estimator that excludes the suspicious instruments, unless the instruments obtained from these moment conditions outnumbers the time-invariant regressors by more than the number of excluded instruments. Even if a difference-in-Hansen test is feasible, we might be confronted with a weak instruments problem under the restricted estimation if we exclude particularly relevant instruments.

The two-stage approach outlined in Section 4 offers a successive testing strategy. At the first stage, specification tests should be carried out to gain confidence in the consistency of the coefficient estimates for the time-varying regressors. Subsequently, such model misspecifications can be excluded under the alternative hypothesis for the Hansen (1982) at the second stage. Based on the two-stage residuals $\hat{\hat{\mathbf{e}}}$,[17] we can then compute the Hansen (1982) test statistic for the validity of the second-stage overidentifying restrictions only:

$$\hat{\hat{\tau}}_\gamma = \left(\frac{1}{\sqrt{N}}\hat{\hat{\mathbf{e}}}'\mathbf{Z}_\gamma\right)\hat{\hat{\Xi}}_v^{-1}\left(\frac{1}{\sqrt{N}}\mathbf{Z}_\gamma'\hat{\hat{\mathbf{e}}}\right). \tag{27}$$

[16] Arellano and Bond (1991) propose a test for serial correlation based on the first-differenced residuals.

[17] Notice that $\hat{\hat{\mathbf{e}}} = \hat{\hat{\mathbf{v}}}$ because the first-stage estimation error drops out when inserting estimates for the unknown population parameters.

With an optimal weighting matrix $\hat{\hat{\Xi}}_v^{-1}$, the test statistic has a limiting χ^2 distribution with $K_{z\gamma} - K_f$ degrees of freedom, where $K_{z\gamma}$ denotes the number of linearly independent second-stage instruments in $\mathbf{Z}_\gamma$. Importantly, $\hat{\hat{\Xi}}_v$ is a consistent estimate of the variance matrix Ξ_v in equation (19) that accounts for the first-stage estimation error.

7 Monte Carlo Simulation

7.1 Simulation Design

We conduct Monte Carlo experiments to analyze the finite sample performance of the two-stage approach in comparison to one-stage GMM estimators. To keep the simulations economical we consider a dynamic panel data model with a single time-varying regressor x_{it} that is correlated with the unobserved unit-specific effects, and one time-invariant regressor f_i that is uncorrelated with them. In practice, the researcher will typically face a larger number of regressors. While the fundamental results should carry over to larger-dimensional models, we note that finite sample distortions of GMM estimators that result from too many overidentifying restrictions might aggravate by adding additional regressors. We generate y_{it} and x_{it} according to the following processes:

$$y_{it} = \lambda y_{i,t-1} + \beta x_{it} + \gamma f_i + \alpha_i + u_{it}, \quad u_{it} \overset{iid}{\sim} \mathcal{N}(0, \sigma_u^2), \tag{28}$$

and

$$x_{it} = \phi x_{i,t-1} + \nu\rho f_i + \nu\sqrt{1-\rho^2}\eta_i + \epsilon_{it}, \quad \epsilon_{it} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2), \tag{29}$$

such that x_{it} is strictly exogenous with respect to u_{it}.[18]

The observed time-invariant variable f_i is obtained as an independent binary variable from a Bernoulli distribution with success probability p. The unobserved unit-specific effects α_i and η_i

[18]Modeling x_{it} as predetermined or endogenous does not affect the qualitative conclusions regarding the coefficient of the time-invariant regressor for appropriately adjusted GMM estimators. It will, however, turn the two-stage QML estimator inconsistent because the first-difference transformation at the first stage requires strict exogeneity.

are generated from a joint normal distribution:

$$\begin{pmatrix} \alpha_i \\ \eta_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\alpha \\ \mu_\eta \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \sigma_{\alpha\eta} \\ \sigma_{\alpha\eta} & p(1-p) \end{pmatrix} \right), \tag{30}$$

such that the variances of η_i and f_i coincide. The particular design of the process for x_{it} guarantees that the correlation between x_{it} and f_i can be altered while keeping the variance of x_{it} unchanged, because

$$Var(x_{it}) = \frac{1}{(1-\phi)^2}\left[\nu^2 p(1-p) + \frac{1-\phi}{1+\phi}\sigma_\epsilon^2\right] \tag{31}$$

is independent of ρ. $\nu \geq 0$ is introduced as a scale parameter. The correlation between x_{it} and f_i is characterized by:

$$Corr(x_{it}, f_i) = \rho\sqrt{\frac{\nu^2 p(1-p)}{\nu^2 p(1-p) + \frac{1-\phi}{1+\phi}\sigma_\epsilon^2}}. \tag{32}$$

Since $\rho \in [-1, 1]$, it can be interpreted as a correlation coefficient net of the variation coming from ϵ_{it}.

We set the long-run coefficient $\beta/(1-\lambda) = 1$ and initialize the processes at $t = -50$ with their long-run means given the realizations of the unit-specific effects:

$$y_{i,-50} = x_{i,-50} + \frac{1}{1-\lambda}\left(\gamma f_i + \alpha_i\right), \tag{33}$$

$$x_{i,-50} = \frac{\nu}{1-\phi}\left(\rho f_i + \sqrt{1-\rho^2}\eta_i\right), \tag{34}$$

and discard the first 50 observations for the estimation. The covariance between the two unobserved fixed effects α_i and η_i is set to $\sigma_{\alpha\eta} = \sigma_\alpha\sqrt{p(1-p)}/2$ which creates a positive correlation between x_{it} and α_i. We also fix $\gamma = 1$, $\sigma_u^2 = 1$, $\nu = 1$, $p = 0.5$ and $\mu_\alpha = \mu_\eta = 0$. To ensure an adequate degree of fit, we obtain the population value of the coefficient of determination for the first-differenced model, $R^2_{\Delta y}$, in a similar fashion as Hsiao et al. (2002). For the data generating process stated above it is given by

$$R^2_{\Delta y} = \frac{\beta^2\sigma_\epsilon^2}{\beta^2\sigma_\epsilon^2 + (1+\phi)(1-\lambda\phi)\sigma_u^2}. \tag{35}$$

We fix $R^2_{\Delta y} = 0.2$ and determine σ^2_ϵ endogenously as

$$\sigma^2_\epsilon = \frac{R^2_{\Delta y}}{1 - R^2_{\Delta y}} \frac{(1+\phi)(1-\lambda\phi)}{\beta^2} \sigma^2_u. \tag{36}$$

Finally, we simulate the data with different combinations for the remaining parameters, namely $\lambda \in \{0.4, 0.8, 0.99\}$, $\sigma^2_\alpha \in \{1, 3\}$, $\phi \in \{0.4, 0.8\}$, and $\rho \in \{0, 0.4\}$. The sample size under consideration is $T \in \{4, 9\}$ and $N \in \{50, 500\}$. In total, we do 3000 repetitions for each simulation.

For the two-stage approach we consider system GMM estimators and the QML estimator of Hsiao et al. (2002) as first-stage estimators. The latter is briefly described in Appendix E. We compare the two-stage QML estimator, "2s-QML", to various GMM estimators that use different sets of instruments and recover the coefficient of the time-invariant regressor either in one or in two stages. First, we set up a system GMM estimator that exploits the full set of moment conditions given in Appendix A and recovers all parameters jointly in one stage, "1s-sGMM (full)".[19] Besides the moment conditions (40) and (44) that result from the presence of the time-invariant regressor, this estimator equals the one proposed by Blundell et al. (2000). To deal with the problems resulting from too many instruments, we set up an alternative system GMM estimator with a collapsed set of instruments, "1s-sGMM (collapsed)".[20] This reduces the number of instruments from 33 to 15 when $T = 4$ and from 143 to 30 when $T = 9$. Furthermore, we consider two-stage variants of both GMM estimators, "2s-sGMM (full)" and "2s-sGMM (collapsed)", respectively. To compute the standard errors of the (first-stage) GMM estimators, we use the robust variance-covariance formula (9) with an unrestricted estimate of Ξ. All GMM estimators are feasible efficient estimators with an initial weighting matrix as chosen by Blundell et al. (2000). We apply the Windmeijer (2005) correction for the standard errors. The second-stage estimates are independent of the choice of the weighting matrix because γ is exactly identified. The corresponding standard errors are based on formula (18) taking into account the first-stage estimation error.

[19] We disregard the moment conditions (41) that are due to homoscedasticity. For the regression constant we exploit only the moment conditions (44) but not the conditions (40).

[20] See Appendix C for the respective transformation matrices.

7.2 Simulation Results

Table 1 summarizes the simulation results for different values of the autoregressive parameter λ holding fixed $\sigma_\alpha^2 = 3$, $\phi = 0.4$, and $\rho = 0.4$. The sample size is small with $T = 4$ and $N = 50$. As a first observation, we recognize that the two-stage approach is very competitive. In particular for the coefficient of the time-invariant regressor it shows a smaller RMSE than the respective one-stage counterpart. We clearly see that the quality of the second-stage estimates hinges crucially on the choice of the first-stage estimator. The large bias of the GMM estimators with the full set of instruments readily transmits into poor second-stage estimates while the two-stage QML estimator convinces us with small biases irrespective of the parameter design.

[Table 1 about here.]

The finite sample bias of GMM estimators that exploit the full set of moment conditions can become tremendous. In the baseline scenario, $\lambda = 0.4$, it reaches 27 percent for the coefficient λ in case of one-stage estimation, and 30 percent for two-stage estimation. The magnitude is similar for the coefficient γ. Reducing the number of instruments with the collapsing procedure yields a strong bias reduction. It shrinks below 3 percent for all coefficients, comparable to the bias of the two-stage QML estimator. The root mean square error (RMSE) shows less clear a picture. While collapsing helps for the coefficient λ, it does not improve the RMSE for β and γ. Particularly for the latter, the reduced bias seems to come at the cost of a larger dispersion. Noteworthy, the RMSE of the two-stage estimator with the full set of instruments is lowest among all estimators under consideration for the coefficient of the time-invariant regressor. However, having a look at the size distortions it is clearly visible that this smaller RMSE does not compensate the poor performance in terms of bias relative to the GMM estimators with the collapsed instruments or the two-stage QML estimator.[21]

The average ratio of the estimated standard errors to the observed standard deviation of the estimators is in most cases reasonably close to unity. An exception are the QML estimates for the coefficient λ when its true value is 0.4. Here, the standard error estimates fall short of the observed standard deviation by about 13 percent. This anomaly can be explained by the observation that

[21] Large size distortions of the Wald test for the system GMM estimator are also documented by Bun and Windmeijer (2010) for the autoregressive parameter.

the QML estimates for λ feature a bimodal distribution with one peak close to the true value of 0.4 and another one close to unity.[22] When we neglect those 44 estimates (out of 3000) that are larger than 0.8, the ratio of the standard errors to the standard deviation jumps up to 1.03. The problematic estimates of the first-stage QML estimator also affect the second-stage estimation of the coefficient γ. When the QML estimates of λ are above 0.8, then the majority of the second-stage estimates of γ even has the wrong sign by falling below zero with a mean at -0.27. Irrespective of this effect, we obtain very promising results for the second-stage standard errors that correct for the first-stage estimation error. On average they are reasonably close to the observed standard deviation.

Increasing the persistence of the data generating process for y_{it} does not produce a clear-cut picture. For the coefficients of the time-varying regressors we obtain strong reductions both of the bias and the RMSE.[23] To the contrary, the GMM results deteriorate for the coefficient of the time-invariant regressor when changing λ from 0.4 to 0.8 and improve again when increasing λ to 0.99. We observe a similar non-uniform behavior for the size statistics with increasing values of λ. The size distortions of the Wald tests for the GMM estimators first become larger when increasing λ from 0.4 to 0.8 but become smaller again when heightening λ further to 0.99. In particular for the GMM estimators with the full set of instruments we notice large overrejections as a consequence of the considerable biases. For the two-stage QML estimator, the bias and RMSE get only slightly worse with higher persistence of the dependent variable.

In Table 2 we present the simulation results for alternative sample sizes and with the same parameterization as in Table 1, holding fixed $\lambda = 0.4$. The findings are not surprising but a few observations shall be mentioned. For the GMM estimator with the full set of instruments both the bias and the RMSE are reduced when we increase the time dimension from 4 to 9 periods, despite the fact that the instruments count goes up from 33 to 143. When the cross-sectional dimension becomes large, $N = 500$, the RMSE turns in favor of the full set of instruments compared to the collapsed one while the latter is still preferred in terms of bias. Independent of the sample size, we find again that the two-stage GMM estimator shows a smaller RMSE than the corresponding one-stage estimator for the coefficient of the time-invariant regressor. For the QML estimator we

[22] Juodis (2013) provides a technical explanation for this identification problem of the transformed likelihood estimator in small samples.

[23] This observation is consistent with the results of Hayakawa (2007).

can observe that the bimodal feature of the distribution disappears with increasing T or N.

[Table 2 about here.]

We also analyze the performance of the estimators under alternative parameterizations of the data generating process. Table 3 presents the results for the three situations of a reduction of the variance σ^2_α of the unit-specific effects from 3 to 1, an increase in the persistence parameter ϕ from 0.4 to 0.8, or an elimination of the correlation between x_{it} and f_i by setting $\rho = 0$, respectively. In the first case, the RMSE is reduced for all parameters. For the coefficient of the lagged dependent variable, the GMM estimators now even become superior to the QML estimator. This result is consistent with previous findings of Binder et al. (2005) and Bun and Windmeijer (2010) that GMM estimators tend to suffer from weak instruments when the variance of the unit-specific effects is large. In the second scenario, the higher persistence of x_{it} yields small improvements for the coefficients of the time-varying regressors. At the same time we observe a sharp deterioration of the results for the coefficient of the time-invariant regressor. The reason is that the latter now explains relatively less of the variation in y_{it} due to the larger variance of the regressor x_{it}. Finally, removing the correlation between the time-varying and the time-invariant regressor leaves the estimates for λ and β virtually unaffected but has a notably positive effect on the precision of the coefficient γ. Concerning the comparison of one-stage and two-stage estimators, the results in Table 3 largely confirm the picture of Table 1. The RMSE of the two-stage estimator is always smaller than that of the corresponding one-stage estimator for the coefficient of the time-invariant regressor while it is the other way round for the coefficients of the time-varying regressors.

[Table 3 about here.]

Importantly, irrespective of the simulation design, when we ignore the first-stage estimation error by assuming $\Xi_v = \Xi_e$ in equation (18), we substantially underestimate the second-stage standard errors. We contrast these estimates in Table 4. For the small sample size with $T = 4$ and $N = 50$ the uncorrected standard errors are on average 10 to 32 percent below the actual standard deviation of the coefficient estimates. Not surprisingly, the underestimation is less severe in the last simulation design where the exogenous time-varying and the time-invariant regressor are uncorrelated because this removes asymptotically the influence of the first-stage estimation error in the coefficient β on the second-stage estimates. A second noteworthy observation is that the standard error correction becomes less relevant when the sample size increases, in particular in the direction of observing more time periods.

[Table 4 about here.]

8 Empirical Application: Distance and FDI

Transportation costs play an important role in theoretical models of bilateral trade and direct investment determination. Empirically, geographical distance has been used extensively as a proxy for transportation costs in confronting gravity models with the data.[24] A major complication in the estimation of such gravity equations with panel data is the time-invariant nature of the geographical distance variable when controlling for unobserved country-specific, industry-specific, or firm-specific effects. While methods for fixed-effects models wipe out all time-invariant characteristics, a pure random-effects model may impose exogeneity assumptions that are too strong to be justifiable. A compromise between the two extremes is the Hausman and Taylor (1981) classification of regressors into subgroups of variables that are correlated with the unobserved effects and those that are not.

Egger and Pfaffermayr (2004a) extend this approach to a seemingly unrelated regressions (SUR) setup to identify the effects of distance on trade and FDI. The authors estimate a static SUR model based on bilateral data at the industry level for the United States and Germany, respectively.[25] They argue that the geographic distance between two countries is correlated with the unobserved time-invariant propensity to invest abroad, for example due to decreasing cultural proximity. Therefore, appropriate instruments need to be deployed. The sum of the real gross domestic product of both countries (henceforth referred to as bilateral GDP), which is used as a predictor of outward FDI, is assumed to be correlated with unobserved trade-partner effects. A measure for the similarity in the country size as well as the factor endowments in physical and human capital are classified as truly exogenous in the sense of Assumption 2 and could thus serve

[24] See Egger and Pfaffermayr (2004a) and the references therein.

[25] The data set is available in the Journal of Applied Econometrics Data Archive. For a variable description, see Egger and Pfaffermayr (2004a). The data is observed on an annual basis for 341 bilateral industry-level relationships between 1989 to 1999. The panel is unbalanced with irregular patterns of missing observations.

as instruments, while that is not the case for relative labor endowment in the FDI equation.[26]

While the SUR approach yields potential efficiency gains, estimating the model equation by equation still results in consistent estimates. We focus here on a re-estimation of the FDI model for the Unites States. In this case, Egger and Pfaffermayr (2004a) find a very large and statistically significant effect of distance, while for Germany and in the bilateral exports model the effect is either relatively small or even statistically insignificant. To assess the robustness of their results, we run a simple specification test for the static model. If there is no serial correlation in the idiosyncratic error term, the errors from the first-differenced equation should exhibit a serial correlation of -0.5.[27] With the data at hand, it is estimated to be -0.113 which is significantly different from -0.5 at the 1% level. This result has several implications. First, standard errors should be made robust to serial correlation in a static fixed-effects regression for valid inference. Second, the generalized least squares (GLS) procedure used by Egger and Pfaffermayr (2004a) to obtain their Hausman and Taylor (1981) estimates is based on an incorrect estimate of the variance matrix. Third and most severe, if the serial correlation is a result of a data generating process that includes a lagged dependent variable, static model estimates potentially yield estimates with sizable biases of short-run and long-run effects as shown by Egger and Pfaffermayr (2004b).[28] Given these arguments, we re-estimate the FDI equation for the United States in a dynamic setting.

The static model estimates based on the within transformation that removes all time-invariant components are replicated in the first column of Table 5. The coefficient estimates are identical to those in the original paper. Yet, we compute standard errors that are robust to heteroscedasticity and serial correlation. They are much higher compared to the conventional standard errors reported by Egger and Pfaffermayr (2004a) such that some of the regressors turn statistically insignificant or significant only at a lower level. The second column is a re-estimation of their single-equation GLS estimates under the Hausman and Taylor (1981) assumptions. Our coefficient estimates differ slightly from the original ones due to differences in the variance component estimates. However,

[26] In the bilateral exports equation, they still treat labor endowments as exogenous based on overidentification tests. However, to the extent that the unobserved time-invariant effects capture similar country-industry characteristics in both equations such an asymmetric treatment is disputable.

[27] See Wooldridge (2002, Chapter 10.6.3) for a description of the test.

[28] Besides this econometric argumentation in favor of a dynamic model specification, the recent literature on FDI determinants also motivates dynamic gravity models to cope with the persistence of bilateral FDI. See for example Kimura and Todo (2010) and Kahouli and Maktouf (2014). Both also employ system GMM estimators but remain silent on the instruments used to identify the coefficients of the time-invariant regressors.

the qualitative conclusions are the same.

[Table 5 about here.]

The dynamic model specification estimated with a system GMM estimator supports the assumption of history dependence in the data generating process of the real bilateral stock of outward FDI. The autoregressive coefficient exceeds 0.8 both with a one-stage and a two-stage estimation strategy.[29] For the two-stage estimator, only 3 out of 56 instruments at the first stage differ from the one-stage estimator. More specifically, we are using first differences instead of levels of the variables that are assumed to be uncorrelated with the unobserved effects according to Assumption 2 (similarity in country size, relative physical capital endowment, and relative human capital endowment) as instruments for the equation in levels because they are partially correlated with the omitted distance variable. For our main variable of interest, the time-invariant geographical distance, the point estimates in both cases are very similar while the standard errors under the two-stage approach are much higher such that the coefficient estimate is no longer statistically significant.

When testing the validity of the dynamic model and instruments used, we find that the Hansen (1982) overidentification test based on the one-stage estimates does not provide evidence for misclassification. We cannot reject the null hypothesis of joint validity of all instruments. The same holds for the first-stage estimation of our two-stage estimator. Contrarily, the test based on the second-stage estimates only rejects the chosen Hausman and Taylor (1981) classification of the variables.[30] The Arellano and Bond (1991) specification test for absence of second-order serial correlation in the first-differenced residuals is easily passed by both estimators.

To address the potential invalidity of the second-stage instruments, we redo the analysis without

[29] The one-stage moment conditions are given in Appendix A, disregarding conditions (40) and (41). To avoid problems of instrument proliferation we only use the second to fourth lag of the dependent variable as instruments in the first-differenced equation and the lags 0 to 4 for the remaining regressors that are assumed to be strictly exogenous with respect to the idiosyncratic disturbances. In addition, we collapse the instrument matrices in accordance with the procedure described in Appendix C. We follow Blundell et al. (2000) to form the initial weighting matrix. For two-stage GMM estimation we treat all time-varying regressors as potentially correlated with the first-stage effects $\tilde{\alpha}_i$, as explained in Section 4. The second-stage moment conditions are given by equation (14), again applying the collapsing procedure. At the second stage, we only report results from a one-step estimator without optimal weighting matrix because the feasible efficient estimator tends to be relatively sensitive when some of the instruments are weak.

[30] For its validity, the Hansen (1982) test statistic needs to be based on an optimal weighting matrix. Since we observed sensitive second-stage coefficient estimates when using an optimal weighting matrix in the presence of weak instruments, this may also undermine the reliability of the Hansen (1982) test. In the current case, the physical capital endowment is such an instrument that is only weakly correlated with distance.

classifying the relative physical capital endowment as an exogenous regressor with respect to the unobserved effects. Its unconditional correlation with the time-invariant distance variable is only 0.01. It is thus of no use to identify the coefficient of the latter. The results are reported in Table 6. We only observe minor changes in the coefficient estimates but the Hansen (1982) test no longer rejects the null hypothesis of joint validity of the remaining instruments (similarity in country size and relative labor endowment, in addition to a constant). Notice in particular that the estimates for the time-varying regressors with the two-stage estimator are entirely unaffected because the first-stage moment conditions remain the same as before.

[Table 6 about here.]

The estimation results hint at the appropriateness of a dynamic instead of a static model. For making the dynamic estimation results comparable with the static estimates, we compute the long-run marginal effect of distance evaluated at the mean of the relative capital-labor ratio (-0.12 in logarithms). In the dynamic model, the short-run effects are given by the marginal effects conditional on the lagged dependent variable while long-run effects are obtained by scaling the short-run effects by the multiplier $(1 - \lambda)^{-1}$. Both in Table 5 and 6 we can see that the implied long-run effect of distance on the real bilateral stock of outward FDI is much smaller in the dynamic model (and insignificant when using the two-stage estimator).

Finally, the correction of the second-stage standard errors as emphasized in Section 4 proves to be important. Table 6 reports the uncorrected standard errors in the final column. For the time-invariant distance variable, it is more than halved without the correction which would signal erroneously statistical significance even at the 1% level. Similar observations can be made for the short-run and long-run marginal effects. At the same time, the Hansen (1982) test would reject the null hypothesis of joint validity of the second-stage instruments at the 10% level if it is based on an uncorrected and therefore no longer optimal weighting matrix.

Overall, the static model estimates by Egger and Pfaffermayr (2004a) tend to strongly overestimate the effect of distance on bilateral FDI due to the ignored persistence of the dependent variable. Moreover, the results from the dynamic model obtained with system GMM estimators remain inconclusive whether the effect is even statistically significantly different from zero.

9 Conclusion

Estimation of linear dynamic panel data models with unobserved unit-specific heterogeneity is a challenging task when the time dimension is short. The identification of the coefficients of time-invariant regressors poses additional complications and requires further assumptions on the orthogonality of the regressors and the unobserved unit-specific effects. These orthogonality assumptions imply additional moment conditions that can be used to form a GMM estimator that estimates all parameters simultaneously. As an alternative we propose a two-stage estimation strategy. At the first stage, we subsume the time-invariant regressors under the unit-specific effects and estimate the coefficients of the time-varying regressors. At the second stage, we regress the first-stage residuals on the time-invariant regressors. Both time-varying and time-invariant variables that are assumed to be uncorrelated with the unit-specific effects qualify as instruments at the second stage. The corresponding overidentifying restrictions can be tested with the usual specification tests at the second stage.

We can base the first-stage regression on any estimator that consistently estimates the coefficients of the time-varying regressors without relying on estimates of the coefficients of time-invariant regressors. In this paper, we discuss GMM-type estimators and a transformed likelihood estimator as potential first-stage candidates. The latter is entirely based on the model in first differences and thus necessarily requires the two-stage approach to identify the coefficients of time-invariant regressors. In general, the two-stage approach is neither restricted to models with a short time dimension nor to dynamic models. It has two main advantages compared to the estimation of all parameters at once. First, the estimation of the coefficients of the time-varying regressors is robust to a model misspecification with regard to the time-invariant variables. Second, the researcher can exploit advantages of first-stage estimators that rely on transformations to eliminate the unit-specific heterogeneity such as first differences or forward orthogonal deviations.

Our Monte Carlo analysis points out that the two-stage approach works very well in finite sample but it crucially hinges upon the choice of the first-stage estimator. Suitable candidates are the QML estimator and GMM estimators that effectively limit the number of overidentifying restrictions. GMM estimators that are based on the full set of available moment conditions are shown to suffer from instrument proliferation even at a modest time span. As a consequence, the resulting first-stage estimation error translates into poor second-stage estimates.

Importantly, the two-stage approach requires an adjustment of the second-stage standard errors due to the additional variation that comes from the first-stage estimation error. We provide the asymptotic variance formula for the second-stage estimator. Our Monte Carlo results demonstrate that the adjustment works well and is quantitatively important. The relevance of the standard error correction is also demonstrated in our empirical application.

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Appendix

A GMM Moment Conditions

In this appendix, we list the model implied moment conditions for one-stage GMM estimation. Following Arellano and Bond (1991) and Blundell et al. (2000), Assumption 1 implies the following $T(T-1)/2$ moment conditions for the model in first differences:

$$E[y_{i,t-s}\Delta u_{it}] = 0, \quad t = 2, 3, \dots, T, \quad 2 \leq s \leq t. \tag{37}$$

Under strict exogeneity of the variables $\mathbf{x}_{it}$ according to Assumption 3 we have another $K_x(T+1)(T-1)$ moment conditions:

$$E[\mathbf{x}_{is}\Delta u_{it}] = \mathbf{0}, \quad t = 2, 3, \dots, T, \quad 0 \leq s \leq T. \tag{38}$$

In the case of predetermined regressors there are only the following $K_x(T+2)(T-1)/2$ moment conditions available:

$$E[\mathbf{x}_{i,t-s}\Delta u_{it}] = \mathbf{0}, \quad t = 2, 3, \dots, T, \quad 1 \leq s \leq t. \tag{39}$$

At this stage, we do not need to make a distinction between regressors that are correlated and those that are uncorrelated with α_i. Following Arellano and Bover (1995), the presence of time-invariant regressors provides another $K_f(T-1)$ moment conditions:

$$E[\mathbf{f}_i\Delta u_{it}] = \mathbf{0}, \quad t = 2, 3, \dots, T. \tag{40}$$

When the disturbances u_{it} are homoscedastic through time, Ahn and Schmidt (1995) suggest another $T-2$ moment conditions:

$$E[y_{i,t-2}\Delta u_{i,t-1} - y_{i,t-1}\Delta u_{it}] = 0, \quad t = 3, \dots, T. \tag{41}$$

We can combine these moment conditions for the first-differenced equation:

$$E[\mathbf{Z}'_{di}\mathbf{D}\mathbf{e}_i] = \mathbf{0}, \tag{42}$$

where $\mathbf{Z}_{di} = (\mathbf{Z}_{dyi}, \mathbf{Z}_{dxi}, \mathbf{I}_{T-1} \otimes \mathbf{f}'_i, \mathbf{Z}_{dui})$ with

$$\mathbf{Z}_{dyi} = \begin{pmatrix} \mathbf{z}'_{dyi2} & 0 & \cdots & 0 \\ 0 & \mathbf{z}'_{dyi3} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{z}'_{dyiT} \end{pmatrix}, \quad \mathbf{Z}_{dxi} = \begin{pmatrix} \mathbf{z}'_{dxi2} & 0 & \cdots & 0 \\ 0 & \mathbf{z}'_{dxi3} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{z}'_{dxiT} \end{pmatrix},$$

$$\mathbf{Z}_{dui} = \begin{pmatrix} y_{i1} & 0 & \cdots & 0 \\ -y_{i2} & y_{i2} & & \vdots \\ 0 & -y_{i,3} & \ddots & 0 \\ \vdots & & \ddots & y_{i,T-2} \\ 0 & \cdots & 0 & -y_{i,T-1} \end{pmatrix}$$

and $\mathbf{z}_{dyit} = (y_{i0}, y_{i1}, \ldots, y_{i,t-2})'$. The instruments $\mathbf{z}_{dxit}$ differ according to the assumption about the regressor variables. We have $\mathbf{z}_{dxit} = (\mathbf{x}'_{i0}, \mathbf{x}'_{i1}, \ldots, \mathbf{x}'_{iT})'$ under strict exogeneity, and $\mathbf{z}_{dxit} = (\mathbf{x}'_{i0}, \mathbf{x}'_{i1}, \ldots, \mathbf{x}'_{i,t-1})'$ for predetermined regressors.

For the regressors $\mathbf{x}_{1it}$, Arellano and Bond (1991) introduce the following $K_{x1}(T+1)$ level moment conditions:

$$E[\mathbf{x}_{1i0}e_{i1}] = \mathbf{0}, \quad \text{and} \quad E[\mathbf{x}_{1it}e_{it}] = \mathbf{0}, \quad t = 1, 2, \ldots, T. \tag{43}$$

Arellano and Bover (1995) further suggest K_{f1} moment conditions for the time-invariant regressors $\mathbf{f}_{1i}$ that are uncorrelated with the unit-specific effects α_i:

$$E\left[\mathbf{f}_{1i}\sum_{t=1}^{T} e_{it}\right] = \mathbf{0}. \tag{44}$$

To add further moment conditions for the model in levels we need to impose the following assumption:

Assumption 5: $E[\Delta y_{i1}\alpha_i] = 0$, and $E[\Delta \mathbf{x}_{2it}\alpha_i] = 0,\ t = 1, 2, \ldots, T$.[31]

Under the additional Assumption 5, Blundell and Bond (1998) establish the following $T-1$ linear moment conditions for the model in levels:

$$E[\Delta y_{i,t-1} e_{it}] = 0, \quad t = 2, 3, \ldots, T. \tag{45}$$

Moreover, Arellano and Bover (1995) and Blundell et al. (2000) introduce another $K_{x2}T$ moment conditions for the regressors $\mathbf{x}_{2it}$ under Assumption 5:

$$E[\Delta \mathbf{x}_{2it} e_{it}] = \mathbf{0}, \quad t = 1, 2, \ldots, T. \tag{46}$$

All remaining moment conditions for the model in levels are redundant.[32] We can now combine the level moment conditions:

$$E[\mathbf{Z}'_{li}\mathbf{e}_i] = \mathbf{0}, \tag{47}$$

where $\mathbf{Z}_{li} = (\mathbf{Z}_{lyi}, \mathbf{Z}_{lxi}, \mathbf{F}_{1i})$, with

$$\mathbf{Z}_{lyi} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \Delta y_{i1} & 0 & \cdots & 0 \\ 0 & \Delta y_{i2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \Delta y_{i,T-1} \end{pmatrix},$$

[31] To guarantee that Δy_{it} and $\Delta \mathbf{x}_{2it}$ are uncorrelated with α_i a restriction on the initial conditions has to be satisfied. Deviations of y_{i0} and $\mathbf{x}_{2i0}$ from their long-run means must be uncorrelated with α_i. A sufficient but not necessary condition for Assumption 5 to hold is joint mean stationarity of the processes y_{it} and $\mathbf{x}_{it}$. Moreover, $E[\Delta y_{it}\alpha_i] = 0,\ t = 2, 3, \ldots, T$, is implied by Assumption 5. See Blundell and Bond (1998), Blundell et al. (2000), and Roodman (2009) for a discussion.

[32] The moment conditions (45) and (46) that result under Assumption 5 do not help identifying $\boldsymbol{\gamma}$ because it is unlikely that these instruments are correlated with the time-invariant regressors. Compare Arellano (2003), Chapter 8.5.4.

and

$$
\mathbf{Z}_{lxi} = \begin{pmatrix} \mathbf{x}'_{1i0} & \mathbf{x}'_{1i1} & 0 & \cdots & 0 & \Delta\mathbf{x}'_{2i1} & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{x}'_{1i2} & & \vdots & 0 & \Delta\mathbf{x}'_{2i2} & & \vdots \\ \vdots & \vdots & & \ddots & 0 & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{x}'_{1iT} & 0 & \cdots & 0 & \Delta\mathbf{x}'_{2iT} \end{pmatrix}.
$$

Ahn and Schmidt (1995) derive an additional nonlinear moment condition under homoscedasticity of u_{it}, namely $E[\bar{u}_i \Delta u_{i2}] = 0$. In this paper, we restrict our attention to the linear moment conditions above.

B Feasible Efficient GMM Estimation

Let $\Omega = E[\mathbf{e}_i \mathbf{e}'_i | \mathbf{Z}_i]$. Under homoscedasticity, $E[u_{it}^2 | \mathbf{Z}_i] = \sigma_u^2$ and $E[\alpha_i^2 | \mathbf{Z}_i] = \sigma_\alpha^2$, and prior knowledge of $\tau = \sigma_\alpha^2 / \sigma_u^2$, an optimal weighting matrix is:

$$
\mathbf{V}_N = N \left[\mathbf{Z}' \tilde{\mathbf{H}} (\mathbf{I}_N \otimes \tilde{\Omega}) \tilde{\mathbf{H}}' \mathbf{Z} \right]^{-1}, \tag{48}
$$

with $\tilde{\Omega} = \tau \boldsymbol{\iota}_T \boldsymbol{\iota}'_T + \mathbf{I}_T$ such that $\mathbf{V} = \sigma_u^2 \Xi^{-1}$. When the estimator only involves moment conditions for the first-differenced equation such that $\tilde{\mathbf{H}}' \mathbf{Z} = \tilde{\mathbf{D}}' \mathbf{Z}_d$, the optimal weighting matrix (48) boils down to $\mathbf{V}_N = N(\mathbf{Z}'_d \tilde{\mathbf{D}} \tilde{\mathbf{D}}' \mathbf{Z}_d)^{-1}$ independent of τ since $\mathbf{D} \tilde{\Omega} \mathbf{D}' = \mathbf{D}\mathbf{D}'$, as discussed by Arellano and Bond (1991).

When τ is unknown or homoscedasticity is too strong an assumption, it is common practice to use a first-step weighting matrix of the following form:

$$
\mathbf{V}_N = N \left[\mathbf{Z}' (\mathbf{I}_N \otimes \Omega^*) \mathbf{Z} \right]^{-1}, \tag{49}
$$

with different choices for Ω^*. Among others, Arellano and Bover (1995) and Blundell and Bond (1998) use $\Omega^* = \mathbf{I}_{2T-1}$, while Blundell et al. (2000) take the first-order serial correlation in the first-differenced residuals into account by choosing

$$
\Omega^* = \begin{pmatrix} \mathbf{D}\mathbf{D}' & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_T \end{pmatrix}.
$$

When σ_α^2 is small, Windmeijer (2000) suggests to use $\Omega^* = \mathbf{H}\mathbf{H}'$. In the latter case, the first-step weighting matrix (49) equals the optimal weighting matrix (48) under $\tau = 0$. A reasonable alternative is the weighting matrix (48) with an adequate choice (or prior estimate) of τ.

As discussed in Section 3, the second-step weighting matrix is formed as $\mathbf{V}_N = \hat{\Xi}^{-1}$. Under homoscedasticity, an estimate of Ξ can be obtained as $\hat{\Xi} = N^{-1} \sum_{i=1}^N \mathbf{Z}_i'\mathbf{H}\hat{\Omega}\mathbf{H}'\mathbf{Z}_i$ with an unrestricted estimate $\hat{\Omega} = N^{-1} \sum_{i=1}^N \hat{\mathbf{e}}_i \hat{\mathbf{e}}_i'$ or a restricted estimate $\hat{\Omega} = \hat{\sigma}_\alpha^2 \boldsymbol{\iota}_T \boldsymbol{\iota}_T' + \hat{\sigma}_u^2 \mathbf{I}_T$. The variance estimates $\hat{\sigma}_\alpha^2$ and $\hat{\sigma}_u^2$ can be obtained as follows:

$$\hat{\sigma}_e^2 = \frac{1}{NT - (1 + K_x + K_f)} \sum_{i=1}^N \sum_{t=1}^T \hat{e}_{it}^2, \tag{50}$$

$$\hat{\sigma}_\alpha^2 = \frac{1}{NT(T-1)/2 - (1 + K_x + K_f)} \sum_{i=1}^N \sum_{t=1}^{T-1} \sum_{s=t+1}^T \hat{e}_{it}\hat{e}_{is}, \tag{51}$$

$$\hat{\sigma}_u^2 = \hat{\sigma}_e^2 - \hat{\sigma}_\alpha^2. \tag{52}$$

C Transformations of GMM Instruments

This appendix provides examples of the transformation matrix $\mathbf{R}$ that are relevant in practical applications.[33] In the following, we restrict our attention to block-diagonal versions of $\mathbf{R}$:

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_l \end{pmatrix},$$

such that $\mathbf{H}'\mathbf{Z}_i\mathbf{R} = (\mathbf{D}'\mathbf{Z}_{di}\mathbf{R}_d, \mathbf{Z}_{li}\mathbf{R}_l)$. Similarly, we consider a block-diagonal partition of the transformation matrix for the first-differenced equation:

$$\mathbf{R}_d = \begin{pmatrix} \mathbf{R}_{dy} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{dx} \otimes \mathbf{I}_{K_x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{df} \otimes \mathbf{I}_{K_f} \end{pmatrix},$$

conformable for multiplication with the instruments matrix $\mathbf{Z}_{di}$ given in Appendix A. For simplicity, we disregard the moment conditions (41) that are based on the homoscedasticity of u_{it}.

[33] Mehrhoff (2009) provides similar transformation matrices for an AR(1) process.

Often, the instrument count is reduced by restricting the number of lags used to construct the instrument matrix. This procedure is equivalent to the construction of a transformation matrix $\mathbf{R}_d$ that selects the appropriate columns of the full matrix $\mathbf{Z}_{di}$. As an example, the following matrices restrict the lag depth to $\kappa \geq 1$ for both the lagged dependent variable $y_{i,t-1}$ and strictly exogenous regressors $\mathbf{x}_{it}$ while also discarding future values of the latter:

$$\mathbf{R}_{dy} = \begin{pmatrix} 1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\kappa 2} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\kappa 3} & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{J}_{\kappa,T-1} \end{pmatrix}, \quad \mathbf{R}_{dx} = \begin{pmatrix} \tilde{\mathbf{J}}_{\kappa 3} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{J}}_{\kappa 4} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \tilde{\mathbf{J}}_{\kappa T} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{J}_{\kappa,T+1} \end{pmatrix},$$

where $\mathbf{J}_{\kappa s} = \mathbf{I}_s$ if $s \leq \kappa$, and $\mathbf{J}_{\kappa s} = (\mathbf{0}, \mathbf{I}_\kappa)'$ with dimension $s \times \kappa$ if $s > \kappa$, and $\tilde{\mathbf{J}}_{\kappa s} = (\mathbf{J}'_{\kappa s}, \mathbf{0})'$ with dimension $(T+1) \times \min\{s, \kappa\}$. We set $\mathbf{R}_{df} = \mathbf{I}_{T-1}$ in this case.

Alternatively, the dimension of the instrument matrix can be reduced by collapsing it into smaller blocks. The following transformation matrices linearly combine the columns of $\mathbf{Z}_{di}$, again for the case of strictly exogenous regressors $\mathbf{x}_{it}$:

$$\mathbf{R}_{dy} = \begin{pmatrix} \mathbf{J}^*_{0,1,T-2} \\ \mathbf{J}^*_{0,2,T-3} \\ \vdots \\ \mathbf{J}^*_{0,T-2,1} \\ \mathbf{I}^*_{T-1} \end{pmatrix}, \quad \mathbf{R}_{dx} = \begin{pmatrix} \mathbf{J}^*_{0,T+1,T-2} \\ \mathbf{J}^*_{1,T+1,T-3} \\ \vdots \\ \mathbf{J}^*_{T-3,T+1,1} \\ \mathbf{J}^*_{T-2,T+1,0} \end{pmatrix},$$

where $\mathbf{J}^*_{s_1,s_2,s_3} = (\mathbf{0}_{s_2 \times s_1}, \mathbf{I}^*_{s_2}, \mathbf{0}_{s_2 \times s_3})$ with dimension $s_2 \times (s_1 + s_2 + s_3)$, and $\mathbf{I}^*_{s_2}$ is the s_2-dimensional mirror identity matrix with ones on the antidiagonal and zeros elsewhere. $\mathbf{Z}_{dyi}\mathbf{R}_{dy}$ now corresponds to the collapsed matrix described by Roodman (2009). As a consequence, the $T(T-1)/2$ moment conditions (37) are replaced by the $T-1$ conditions $E\left[\sum_{t=s}^{T} y_{i,t-s} \Delta u_{it}\right] = 0, \ s = 2, 3, \dots, T$. Similarly, the information contained in the $K_x(T+1)(T-1)$ moment conditions (38) is condensed into $K_x(2T-1)$ conditions. The instrument block containing $\mathbf{f}_i$ can be collapsed by setting $\mathbf{R}_{df} = \boldsymbol{\iota}_{T-1}$. The implied K_f moment conditions are $E[\mathbf{f}_i(u_{iT} - u_{i1})] = \mathbf{0}$ instead of the $K_f(T-1)$

conditions (40). The transformation matrices can be further adjusted to combine the collapsing approach with the lag depth restriction.

The instruments for the level equation, for clarity ignoring the moment conditions $E[\mathbf{x}_{1i0}e_{i1}] = \mathbf{0}$, can be collapsed into a set of standard instruments by applying the following transformation:

$$\mathbf{R}_l = \begin{pmatrix} \boldsymbol{\iota}_{T-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\iota}_T \otimes \mathbf{I}_{K_{x1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\iota}_T \otimes \mathbf{I}_{K_{x2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{K_{f1}} \end{pmatrix},$$

such that $\mathbf{Z}_{li}\mathbf{R}_l = [(0, \Delta\mathbf{y}'_{i,(-1)})', \mathbf{X}_{1i}, \mathbf{D}\mathbf{X}_{2i}, \mathbf{F}_{1i}]$.

D Two-Stage GMM Estimation

Consider a first-stage system GMM estimator $\hat{\boldsymbol{\theta}}$ that satisfies the moment conditions $E[\mathbf{Z}'_i\mathbf{H}\tilde{\mathbf{e}}_i] = \mathbf{0}$ for the first-stage model (10), possibly making use of moment conditions for the level equation. Compared to one-stage system GMM estimators, this requires an appropriate adjustment of the instruments $\mathbf{Z}_{li}$ that now have to be uncorrelated with $\tilde{\alpha}_i$ instead of α_i. The instruments $\mathbf{Z}_{di}$ for the transformed model can be left unchanged because $\mathbf{D}\mathbf{e}_i = \mathbf{D}\tilde{\mathbf{e}}_i$. With the notation of Section 5, we obtain the first-stage estimator $\hat{\boldsymbol{\theta}}$ by adapting equation (23), partialling out the intercept term $\bar{\alpha}$:

$$\hat{\boldsymbol{\theta}} = (\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{W}^*)^{-1}\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{y}^*, \tag{53}$$

where $\mathbf{M}_\iota = \mathbf{I}_{K_z} - \boldsymbol{\iota}^*(\boldsymbol{\iota}^{*\prime}\boldsymbol{\iota}^*)^{-1}\boldsymbol{\iota}^{*\prime}$ with $\boldsymbol{\iota}^* = \mathbf{L}'\mathbf{Z}'\tilde{\mathbf{H}}\boldsymbol{\iota}_{NT}$. From equation (53) we can infer an expression for the corresponding influence function $\boldsymbol{\psi}_i$ that is needed to obtain an estimate of $\Xi_{\theta e}$ at the second stage:

$$\boldsymbol{\psi}_i = (\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{W}^*)^{-1}\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{L}'\mathbf{Z}'_i\mathbf{H}\tilde{\mathbf{e}}_i, \tag{54}$$

such that

$$\hat{\hat{\Xi}}_{\theta e} = (\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{W}^*)^{-1}\mathbf{W}^{*\prime}\mathbf{M}_\iota\mathbf{L}'\left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{Z}'_i\mathbf{H}\hat{\tilde{\mathbf{e}}}_i\hat{\hat{\mathbf{e}}}'_i\mathbf{Z}_{\gamma i}\right), \tag{55}$$

where $\hat{\hat{\mathbf{e}}}_i = \mathbf{y}_i - \mathbf{W}_i\hat{\boldsymbol{\theta}} - \hat{\hat{\alpha}}\boldsymbol{\iota}_T$. Notice that plim $N^{-1}\sum_{i=1}^N \mathbf{Z}_i'\mathbf{H}\tilde{\mathbf{e}}_i\mathbf{e}_i'\mathbf{Z}_{\gamma i} = \mathbf{0}$ in the special case where $\mathbf{H} = \mathbf{D}$, the errors are independent and homoscedastic across units and time, and the second-stage instruments $\mathbf{Z}_{\gamma i}$ are time-invariant. Hence, in this particular case $\Xi_{\theta e} = \mathbf{0}$, and ignoring the first-stage estimation error results in an underestimation of the standard errors at the second stage.

E Two-Stage QML Estimation

If $K_{x2} = K_{f2} = 0$ we can immediately estimate model (1) with the random effects maximum likelihood estimator of Bhargava and Sargan (1983) and Hsiao et al. (2002). When this strong assumption does not hold, Hsiao et al. (2002) propose to estimate the coefficients of the time-varying regressors based on the first-differenced model:

$$\Delta y_{it} = \lambda\Delta y_{i,t-1} + \Delta\mathbf{x}_{it}'\boldsymbol{\beta} + \Delta u_{it}, \tag{56}$$

for the time periods $t = 2, 3, \ldots, T$. However, this procedure not only eliminates the incidental parameters α_i but also the time-invariant variables $\mathbf{f}_i$. The latter can be recovered with the two-stage approach described in Section 4.

Hsiao et al. (2002) derive the joint density of $\Delta\tilde{\mathbf{y}}_i = (\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{iT})'$ conditional on the strictly exogenous variables $\Delta\tilde{\mathbf{X}}_i = (\Delta\mathbf{x}_{i1}, \Delta\mathbf{x}_{i2}, \ldots, \Delta\mathbf{x}_{iT})'$. Because Δy_{i0} is unobserved, the marginal density of the initial observations Δy_{i1} conditional on $\Delta\tilde{\mathbf{X}}_i$ cannot be obtained immediately from model (56). Instead, Hsiao et al. (2002) apply linear projection techniques to derive the following expression for the initial observations based on an additional stationarity assumption for the regressors $\mathbf{x}_{it}$:

$$\Delta y_{i1} = b + \sum_{s=1}^T \Delta\mathbf{x}_{is}'\boldsymbol{\pi}_s + \xi_{i1}, \tag{57}$$

with $E[\xi_{i1}|\Delta\tilde{\mathbf{X}}_i] = 0$, $E[\xi_{i1}^2] = \sigma_\xi^2$, $E[\xi_{i1}\Delta u_{i2}] = -\sigma_u^2$, and $E[\xi_{i1}\Delta u_{it}] = 0$ for $t = 3, 4, \ldots, T$. The $1 + K_xT$ coefficients $\boldsymbol{\pi} = (b, \boldsymbol{\pi}_1', \boldsymbol{\pi}_2', \ldots, \boldsymbol{\pi}_T')'$ are additional nuisance parameters that need to be estimated jointly with the parameters of interest. Under homoscedasticity, the variance-covariance matrix of $\Delta\tilde{\mathbf{u}}_i = (\xi_{i1}, \Delta u_{i2}, \ldots, \Delta u_{iT})'$ is given by[34]

$$E[\Delta\tilde{\mathbf{u}}_i \Delta\tilde{\mathbf{u}}_i'] = \sigma_u^2 \ddot{\Omega} = \sigma_u^2 \begin{pmatrix} \omega & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & & \\ \vdots & & & \ddots & -1 \\ 0 & & & -1 & 2 \end{pmatrix},$$

where $\omega = \sigma_\xi^2/\sigma_u^2$. The likelihood function can now be set up for the transformed model $\Delta\tilde{\mathbf{y}}_i = \Delta\tilde{\mathbf{W}}_i\boldsymbol{\theta} + \Delta\tilde{\mathbf{X}}_i\boldsymbol{\pi} + \Delta\tilde{\mathbf{u}}_i$, where

$$\Delta\tilde{\mathbf{W}}_i = \begin{pmatrix} 0 & \mathbf{0} \\ \Delta\mathbf{y}_{i,(-1)} & \Delta\mathbf{X}_i \end{pmatrix}, \quad \Delta\tilde{\mathbf{X}}_i = \begin{pmatrix} 1 & \Delta\mathbf{x}_{i1}' & \Delta\mathbf{x}_{i2}' & \ldots & \Delta\mathbf{x}_{iT}' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{pmatrix}.$$

Decompose $\ddot{\Omega}^{-1} = \mathbf{A}'\mathbf{B}^{-1}\mathbf{A}$, where $\mathbf{A}$ is a $T \times T$ lower-triangular and $\mathbf{B}$ a diagonal matrix.[35] Moreover, let $\mathbf{P} = \mathbf{I}_N \otimes (\mathbf{B}^{-1/2}\mathbf{A})$. The QML estimator for $\boldsymbol{\theta}$ is then given by:

$$\hat{\boldsymbol{\theta}} = (\Delta\tilde{\mathbf{W}}'\hat{\mathbf{P}}'\hat{\mathbf{M}}_x\hat{\mathbf{P}}\Delta\tilde{\mathbf{W}})^{-1}\Delta\tilde{\mathbf{W}}'\hat{\mathbf{P}}'\hat{\mathbf{M}}_x\hat{\mathbf{P}}\Delta\tilde{\mathbf{y}}, \tag{58}$$

where $\hat{\mathbf{M}}_x = \mathbf{I}_{NT} - \hat{\mathbf{P}}\Delta\tilde{\mathbf{X}}(\Delta\tilde{\mathbf{X}}'\hat{\mathbf{P}}'\hat{\mathbf{P}}\Delta\tilde{\mathbf{X}})^{-1}\Delta\tilde{\mathbf{X}}'\hat{\mathbf{P}}'$, and $\hat{\mathbf{P}}$ is a function of the variance estimate $\hat{\omega}$. The variance-covariance matrix of $\hat{\boldsymbol{\theta}}$ is the corresponding partition of the inverse negative Hessian matrix:

$$\Sigma_\theta = (\Delta\tilde{\mathbf{W}}'\mathbf{P}'\mathbf{M}_x\mathbf{P}\Delta\tilde{\mathbf{W}})^{-1}. \tag{59}$$

In our Monte Carlos simulations in Section 7 we obtain the estimate $\hat{\omega}$ by maximizing the concentrated log-likelihood function in terms of ω only, given the analytical first-order conditions for the remaining parameters. The initial values for the QML optimization are obtained in the following steps. First, we obtain consistent system GMM estimates of λ and β, and a variance estimate of σ_u^2 from the corresponding first-differenced residuals. The nuisance parameters $\boldsymbol{\pi}$ are obtained as ordinary least squares estimates from the initial observations equation (57). Second,

[34] Hayakawa and Pesaran (2015) extend the transformed likelihood estimator to accommodate for heteroscedastic errors.

[35] See Hsiao et al. (2002) for details.

given those estimates we evaluate the first-order condition for the variance parameter ω. Third, we update the estimates of the other parameters based on their respective optimality conditions given this estimate of ω. Finally, we repeat the second and third step one more time to obtain a faster convergence of the subsequent Newton-Raphson algorithm.

The second-stage estimator $\hat{\hat{\gamma}}$ for the coefficients of the time-invariant regressors is given by equation (16), and the joint asymptotic distribution of the first-stage and second-stage estimators follows from Proposition 1. Finally, the influence function of the whole parameter vector including the ancillary parameters is given by the inverse negative Hessian matrix multiplied by the score function for unit i. The influence function for the relevant parameter vector $\hat{\boldsymbol{\theta}}$ is then the corresponding partition.

Table 1: Simulation results under different parameterization of λ

Coefficient	Design	Estimator	Bias	RMSE	Size	SE/SD
λ	$\lambda = .4$	1s-sGMM (full)	0.2718	0.1618	0.2250	0.9417
		2s-sGMM (full)	0.3046	0.1703	0.2537	0.9439
		1s-sGMM (collapsed)	-0.0135	0.1432	0.0797	0.9742
		2s-sGMM (collapsed)	-0.0051	0.1450	0.0870	0.9709
		2s-QML	0.0199	0.1247	0.0613	0.8697
	$\lambda = .8$	1s-sGMM (full)	0.0977	0.0958	0.4320	0.9372
		2s-sGMM (full)	0.1036	0.0988	0.4653	0.9432
		1s-sGMM (collapsed)	0.0209	0.0796	0.1327	0.9393
		2s-sGMM (collapsed)	0.0241	0.0805	0.1383	0.9402
		2s-QML	0.0022	0.0708	0.0493	0.9691
	$\lambda = .99$	1s-sGMM (full)	0.0027	0.0038	0.2763	0.9644
		2s-sGMM (full)	0.0029	0.0039	0.2960	0.9760
		1s-sGMM (collapsed)	0.0012	0.0036	0.1220	0.9612
		2s-sGMM (collapsed)	0.0012	0.0037	0.1307	0.9601
		2s-QML	0.0000	0.0037	0.0500	0.9921
β	$\lambda = .4$	1s-sGMM (full)	0.0555	0.1314	0.0757	1.0076
		2s-sGMM (full)	0.0649	0.1328	0.0747	1.0113
		1s-sGMM (collapsed)	0.0209	0.1348	0.0627	0.9853
		2s-sGMM (collapsed)	0.0232	0.1350	0.0637	0.9891
		2s-QML	0.0098	0.1103	0.0537	0.9833
	$\lambda = .8$	1s-sGMM (full)	0.0310	0.0182	0.0780	1.0240
		2s-sGMM (full)	0.0338	0.0183	0.0820	1.0255
		1s-sGMM (collapsed)	0.0125	0.0190	0.0697	0.9944
		2s-sGMM (collapsed)	0.0142	0.0190	0.0683	1.0001
		2s-QML	0.0045	0.0157	0.0520	0.9857
	$\lambda = .99$	1s-sGMM (full)	0.0010	0.0001	0.0683	1.0106
		2s-sGMM (full)	0.0011	0.0001	0.0677	1.0191
		1s-sGMM (collapsed)	0.0007	0.0001	0.0677	1.0034
		2s-sGMM (collapsed)	0.0008	0.0001	0.0677	1.0082
		2s-QML	0.0001	0.0000	0.0540	0.9906
γ	$\lambda = .4$	1s-sGMM (full)	-0.2651	0.5998	0.1403	0.9996
		2s-sGMM (full)	-0.3061	0.5725	0.1667	1.0134
		1s-sGMM (collapsed)	-0.0139	0.6655	0.0737	1.0017
		2s-sGMM (collapsed)	0.0103	0.6331	0.0713	0.9971
		2s-QML	-0.0020	0.5987	0.0733	0.9730
	$\lambda = .8$	1s-sGMM (full)	-0.4401	0.6723	0.2763	0.9738
		2s-sGMM (full)	-0.4754	0.6562	0.3287	0.9865
		1s-sGMM (collapsed)	-0.1145	0.7109	0.1213	0.9687
		2s-sGMM (collapsed)	-0.1027	0.6810	0.1223	0.9718
		2s-QML	0.0100	0.6820	0.0753	0.9903
	$\lambda = .99$	1s-sGMM (full)	-0.2469	0.6172	0.0873	1.0422
		2s-sGMM (full)	-0.2812	0.5848	0.0950	1.0457
		1s-sGMM (collapsed)	-0.1064	0.6643	0.0630	1.0352
		2s-sGMM (collapsed)	-0.1123	0.6356	0.0613	1.0177
		2s-QML	0.0308	0.6926	0.0363	1.0217

Fixed parameters: $\beta = 1 - \lambda$, $\gamma = 1$, $\sigma_\alpha^2 = 3$, $\phi = 0.4$, $\rho = 0.4$, $T = 4$, $N = 50$.
Note: We abbreviate the estimators as follows: "1s" and "2s" refer to one-stage and two-stage estimators, respectively. "QML" is the estimator of Hsiao et al. (2002), and "sGMM" refers to feasible efficient system GMM estimators. We follow Blundell et al. (2000) to form the initial weighting matrix. In parenthesis, we refer to the set of instruments. The bias is measured relative to the true parameter value. RMSE is the root mean square error. The size statistic refers to the actual rejection rate of Wald tests that the parameter estimates equal their true value given a nominal size of 5%. SE/SD is the average standard error relative to the standard deviation of the estimator for the 3000 replications. GMM standard errors are based on formula (9) with an unrestricted estimate of Ξ and the Windmeijer (2005) correction. Second-stage standard errors are based on formula (18).

Table 2: Simulation results for different sample sizes

Coefficient	Design	Estimator	Bias	RMSE	Size	SE/SD
λ	$T=9$	1s-sGMM (full)	0.1957	0.1013	0.1877	1.1044
	$N=50$	2s-sGMM (full)	0.2214	0.1090	0.2533	1.0964
		1s-sGMM (collapsed)	-0.0232	0.0714	0.0647	1.0088
		2s-sGMM (collapsed)	-0.0226	0.0712	0.0650	1.0146
		2s-QML	-0.0034	0.0472	0.0417	1.0245
	$T=4$	1s-sGMM (full)	0.0215	0.0381	0.0630	0.9721
	$N=500$	2s-sGMM (full)	0.0241	0.0392	0.0627	0.9694
		1s-sGMM (collapsed)	0.0012	0.0430	0.0593	0.9891
		2s-sGMM (collapsed)	0.0017	0.0433	0.0583	0.9908
		2s-QML	0.0024	0.0334	0.0477	0.9980
	$T=9$	1s-sGMM (full)	0.0162	0.0189	0.0713	0.9875
	$N=500$	2s-sGMM (full)	0.0188	0.0197	0.0800	0.9824
		1s-sGMM (collapsed)	-0.0010	0.0215	0.0547	0.9833
		2s-sGMM (collapsed)	-0.0007	0.0216	0.0547	0.9824
		2s-QML	-0.0003	0.0153	0.0527	0.9938
β	$T=9$	1s-sGMM (full)	0.0252	0.0830	0.0220	1.2300
	$N=50$	2s-sGMM (full)	0.0323	0.0827	0.0283	1.2122
		1s-sGMM (collapsed)	0.0017	0.0801	0.0687	0.9702
		2s-sGMM (collapsed)	0.0019	0.0804	0.0677	0.9690
		2s-QML	-0.0013	0.0613	0.0510	0.9901
	$T=4$	1s-sGMM (full)	0.0042	0.0355	0.0557	0.9917
	$N=500$	2s-sGMM (full)	0.0053	0.0357	0.0553	0.9875
		1s-sGMM (collapsed)	0.0017	0.0391	0.0563	0.9812
		2s-sGMM (collapsed)	0.0020	0.0392	0.0583	0.9797
		2s-QML	-0.0001	0.0341	0.0527	0.9930
	$T=9$	1s-sGMM (full)	0.0025	0.0212	0.0510	1.0050
	$N=500$	2s-sGMM (full)	0.0032	0.0213	0.0533	1.0032
		1s-sGMM (collapsed)	0.0002	0.0222	0.0530	1.0045
		2s-sGMM (collapsed)	0.0002	0.0222	0.0507	1.0043
		2s-QML	-0.0003	0.0191	0.0557	1.0045
γ	$T=9$	1s-sGMM (full)	-0.2185	0.5134	0.0820	1.0764
	$N=50$	2s-sGMM (full)	-0.2290	0.4817	0.0907	1.0520
		1s-sGMM (collapsed)	-0.0413	0.5689	0.0580	1.0107
		2s-sGMM (collapsed)	0.0181	0.5358	0.0540	0.9978
		2s-QML	0.0026	0.5158	0.0547	0.9861
	$T=4$	1s-sGMM (full)	-0.0281	0.1902	0.0717	0.9693
	$N=500$	2s-sGMM (full)	-0.0313	0.1853	0.0640	0.9929
		1s-sGMM (collapsed)	-0.0131	0.1961	0.0597	0.9908
		2s-sGMM (collapsed)	-0.0093	0.1925	0.0567	0.9959
		2s-QML	-0.0093	0.1803	0.0523	1.0051
	$T=9$	1s-sGMM (full)	-0.0158	0.1759	0.0603	0.9771
	$N=500$	2s-sGMM (full)	-0.0183	0.1636	0.0577	0.9888
		1s-sGMM (collapsed)	-0.0126	0.1718	0.0557	0.9844
		2s-sGMM (collapsed)	0.0012	0.1684	0.0513	0.9838
		2s-QML	0.0011	0.1641	0.0463	0.9865

Fixed parameters: $\lambda = 0.4$, $\beta = 1 - \lambda$, $\gamma = 1$, $\sigma_\alpha^2 = 3$, $\phi = 0.4$, $\rho = 0.4$.
Note: We abbreviate the estimators as follows: "1s" and "2s" refer to one-stage and two-stage estimators, respectively. "QML" is the estimator of Hsiao et al. (2002), and "sGMM" refers to feasible efficient system GMM estimators. We follow Blundell et al. (2000) to form the initial weighting matrix. In parenthesis, we refer to the set of instruments. The bias is measured relative to the true parameter value. RMSE is the root mean square error. The size statistic refers to the actual rejection rate of Wald tests that the parameter estimates equal their true value given a nominal size of 5%. SE/SD is the average standard error relative to the standard deviation of the estimator for the 3000 replications. GMM standard errors are based on formula (9) with an unrestricted estimate of Ξ and the Windmeijer (2005) correction. Second-stage standard errors are based on formula (18).

Table 3: Simulation results under alternative scenarios

Coefficient	Design	Estimator	Bias	RMSE	Size	SE/SD
λ	$\sigma_\alpha^2 = 1$	1s-sGMM (full)	0.0679	0.1088	0.0837	0.9971
	$\phi = .4$	2s-sGMM (full)	0.1132	0.1151	0.1070	0.9987
	$\rho = .4$	1s-sGMM (collapsed)	-0.0312	0.1249	0.0730	0.9902
		2s-sGMM (collapsed)	-0.0216	0.1271	0.0763	0.9887
		2s-QML	0.0199	0.1245	0.0613	0.8706
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	0.2322	0.1410	0.2170	0.9532
	$\phi = .8$	2s-sGMM (full)	0.2553	0.1465	0.2450	0.9486
	$\rho = .4$	1s-sGMM (collapsed)	0.0024	0.1339	0.0813	0.9707
		2s-sGMM (collapsed)	0.0109	0.1355	0.0893	0.9682
		2s-QML	0.0110	0.1196	0.0613	0.8547
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	0.2729	0.1617	0.2293	0.9422
	$\phi = .4$	2s-sGMM (full)	0.2988	0.1693	0.2463	0.9442
	$\rho = 0$	1s-sGMM (collapsed)	-0.0127	0.1438	0.0803	0.9723
		2s-sGMM (collapsed)	-0.0060	0.1454	0.0850	0.9711
		2s-QML	0.0198	0.1244	0.0613	0.8718
β	$\sigma_\alpha^2 = 1$	1s-sGMM (full)	0.0358	0.1226	0.0693	0.9926
	$\phi = .4$	2s-sGMM (full)	0.0489	0.1243	0.0743	0.9990
	$\rho = .4$	1s-sGMM (collapsed)	0.0196	0.1307	0.0673	0.9778
		2s-sGMM (collapsed)	0.0224	0.1315	0.0680	0.9798
		2s-QML	0.0098	0.1103	0.0537	0.9833
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	0.0499	0.1265	0.0747	0.9926
	$\phi = .8$	2s-sGMM (full)	0.0602	0.1277	0.0793	0.9943
	$\rho = .4$	1s-sGMM (collapsed)	0.0327	0.1372	0.0667	0.9840
		2s-sGMM (collapsed)	0.0364	0.1375	0.0677	0.9871
		2s-QML	0.0056	0.1087	0.0527	0.9851
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	0.0577	0.1316	0.0757	1.0072
	$\phi = .4$	2s-sGMM (full)	0.0578	0.1320	0.0743	1.0085
	$\rho = 0$	1s-sGMM (collapsed)	0.0215	0.1347	0.0677	0.9866
		2s-sGMM (collapsed)	0.0222	0.1348	0.0653	0.9900
		2s-QML	0.0098	0.1103	0.0537	0.9832
γ	$\sigma_\alpha^2 = 1$	1s-sGMM (full)	-0.0714	0.4281	0.0800	1.0191
	$\phi = .4$	2s-sGMM (full)	-0.1234	0.3978	0.0890	1.0342
	$\rho = .4$	1s-sGMM (collapsed)	0.0062	0.4704	0.0727	1.0112
		2s-sGMM (collapsed)	0.0182	0.4528	0.0637	1.0028
		2s-QML	-0.0122	0.4451	0.0703	0.9505
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	-0.3876	0.7137	0.1590	1.0185
	$\phi = .8$	2s-sGMM (full)	-0.4444	0.6973	0.2010	1.0352
	$\rho = .4$	1s-sGMM (collapsed)	-0.0590	0.7748	0.0793	1.0010
		2s-sGMM (collapsed)	-0.0512	0.7450	0.0753	1.0105
		2s-QML	-0.0015	0.6949	0.0710	0.9717
	$\sigma_\alpha^2 = 3$	1s-sGMM (full)	-0.1712	0.5224	0.1020	1.0046
	$\phi = .4$	2s-sGMM (full)	-0.1947	0.4856	0.1247	1.0112
	$\rho = 0$	1s-sGMM (collapsed)	-0.0054	0.6154	0.0643	1.0035
		2s-sGMM (collapsed)	0.0192	0.5777	0.0593	0.9982
		2s-QML	0.0073	0.5545	0.0633	0.9854

Fixed parameters: $\lambda = 0.4$, $\beta = 1 - \lambda$, $\gamma = 1$, $T = 4$, $N = 50$.

Note: We abbreviate the estimators as follows: "1s" and "2s" refer to one-stage and two-stage estimators, respectively. "QML" is the estimator of Hsiao et al. (2002), and "sGMM" refers to feasible efficient system GMM estimators. We follow Blundell et al. (2000) to form the initial weighting matrix. In parenthesis, we refer to the set of instruments. The bias is measured relative to the true parameter value. RMSE is the root mean square error. The size statistic refers to the actual rejection rate of Wald tests that the parameter estimates equal their true value given a nominal size of 5%. SE/SD is the average standard error relative to the standard deviation of the estimator for the 3000 replications. GMM standard errors are based on formula (9) with an unrestricted estimate of Ξ and the Windmeijer (2005) correction. Second-stage standard errors are based on formula (18).

Table 4: Corrected versus uncorrected second-stage standard errors

Coefficient	Design		Estimator	Corrected SE/SD	Uncorrected SE/SD
γ	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	1.0134	0.8080
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9971	0.7975
	$T = 4$	$N = 50$	2s-QML	0.9730	0.8325
	$\lambda = 0.8$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	0.9865	0.7094
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9718	0.6899
	$T = 4$	$N = 50$	2s-QML	0.9903	0.7463
	$\lambda = 0.99$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	1.0457	0.8495
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	1.0177	0.7914
	$T = 4$	$N = 50$	2s-QML	1.0217	0.7959
γ	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	1.0520	0.9598
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9978	0.9320
	$T = 9$	$N = 50$	2s-QML	0.9861	0.9542
	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	0.9929	0.8661
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9959	0.8367
	$T = 4$	$N = 500$	2s-QML	1.0051	0.8932
	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	0.9888	0.9549
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9838	0.9358
	$T = 9$	$N = 500$	2s-QML	0.9865	0.9601
γ	$\lambda = 0.4$	$\sigma_\alpha^2 = 1$	2s-sGMM (full)	1.0520	0.7557
	$\phi = 0.4$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9978	0.7046
	$T = 4$	$N = 50$	2s-QML	0.9861	0.7012
	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	0.9929	0.7411
	$\phi = 0.8$	$\rho = 0.4$	2s-sGMM (collapsed)	0.9959	0.6801
	$T = 4$	$N = 50$	2s-QML	1.0051	0.7355
	$\lambda = 0.4$	$\sigma_\alpha^2 = 3$	2s-sGMM (full)	0.9888	0.8820
	$\phi = 0.4$	$\rho = 0$	2s-sGMM (collapsed)	0.9838	0.8753
	$T = 4$	$N = 50$	2s-QML	0.9865	0.8994

Note: See notes to Tables 1 to 3 for a description of the estimators. We report the average standard error relative to its standard deviation for the 3000 replications. Corrected second-stage standard errors are based on formula (18), while uncorrected standard errors ignore the first-stage estimation error.

Table 5: Estimation results: dynamic gravity model (original instruments)

$\ln(\text{outward FDI})_{it}$	Within	HT-GLS	1s-sGMM	2s-sGMM
$\ln(\text{outward FDI})_{i,t-1}$			0.853	0.886
			(0.063)***	(0.064)***
$\ln(\text{distance})_i$		15.434	0.920	0.888
		(2.575)***	(0.449)**	(0.552)
$\ln(\text{distance})_i$ ×	-1.759	-1.744	-0.738	-0.113
$\ln(\text{relative capital-labor ratio})_{it}$	(0.880)**	(0.600)***	(0.320)**	(0.154)
$\ln(\text{bilateral GDP})_{it}$	5.193	5.653	0.782	1.537
	(0.956)***	(0.828)***	(0.562)	(0.747)**
$\ln(\text{bilateral GDP})_{it}$ ×	0.026	0.022	-0.009	-0.006
$\lvert\ln(\text{relative physical capital endowment})_{it}\rvert$	(0.017)	(0.011)*	(0.008)	(0.008)
$\ln(\text{similarity in country size})_{it}$	1.607	1.762	-0.080	0.581
	(0.629)**	(0.355)***	(0.264)	(0.254)**
$\ln(\text{relative physical capital endowment})_{it}$	14.730	14.761	6.499	1.609
	(7.694)*	(5.203)***	(2.769)**	(1.493)
$\ln(\text{relative human capital endowment})_{it}$	0.278	0.276	0.001	-0.054
	(0.207)	(0.148)*	(0.136)	(0.133)
$\ln(\text{relative labor endowment})_{it}$	-12.897	-12.615	-6.281	-0.805
	(7.494)*	(5.289)**	(2.724)**	(1.352)
Constant		-281.250	-28.082	-47.994
		(38.170)***	(15.672)*	(16.600)***
Year dummies	1990–1999	1990–1999	1991–1999	1991–1999
Observations	2,767	2,767	2,198	2,198
Units	341	341	337	337
1st stage				
Instruments			56	56
Arellano-Bond			z = -0.01	z = -0.01
			(0.989)	(0.995)
Hansen			$\chi^2_{37} = 36.85$	$\chi^2_{38} = 39.91$
			(0.476)	(0.385)
2nd stage				
Instruments				4
Hansen				$\chi^2_2 = 17.17$
				(0.000)***
Short-run marginal effect of $\ln(\text{distance})_i$			1.010	0.901
evaluated at the variable mean			(0.465)**	(0.554)
Long-run marginal effect of $\ln(\text{distance})_i$		15.646	6.888	7.931
evaluated at the variable mean		(2.581)***	(3.741)*	(6.720)

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Note: See Egger and Pfaffermayr (2004a) for a data description. We abbreviate the estimators as follows: "Within" denotes the least squares dummy variables estimator, and "HT-GLS" refers to the Hausman and Taylor (1981) generalized least squares estimator. "1s" and "2s" denote one-stage and two-stage estimators, respectively, and "sGMM" refers to system GMM estimators. We follow Blundell et al. (2000) to form the initial weighting matrix for feasible efficient estimation with "1s-sGMM" and the first stage of "2s-sGMM". We collapse the instrument matrices and for the equation in first differences we use the lags 2 to 4 of the dependent variable and the lags 0 to 4 of all other time-varying regressors as instruments. GMM standard errors are based on formula (9) with an unrestricted estimate of Ξ and the Windmeijer (2005) correction. Second-stage standard errors are based on formula (18). The standard errors are reported in parenthesis. All regressions include time dummies. The exogenous variables according to Assumption 2 are the similarity in country size, the relative physical capital endowment, and the relative human capital endowment. "Arellano-Bond" refers to the Arellano and Bond (1991) test for second-order serial correlation in the first-differenced residuals, and "Hansen" to the Hansen (1982) test of the overidentifying restrictions, with the p-values in parenthesis.

Table 6: Estimation results: dynamic gravity model (adjusted instruments)

ln(outward FDI)$_{it}$	HT-GLS	1s-sGMM	2s-sGMM	(uncorrected)
ln(outward FDI)$_{i,t-1}$		0.834	0.886	
		(0.061)***	(0.064)***	
ln(distance)$_i$	15.064	0.807	0.925	
	(3.461)***	(0.376)**	(0.565)	(0.225)***
ln(distance)$_i$ ×	-1.724	-0.690	-0.113	
ln(relative capital-labor ratio)$_{it}$	(0.616)***	(0.291)**	(0.154)	
ln(bilateral GDP)$_{it}$	5.658	0.991	1.537	
	(0.865)***	(0.631)	(0.747)**	
ln(bilateral GDP)$_{it}$ ×	0.022	-0.009	-0.006	
|ln(relative physical capital endowment)$_{it}$|	(0.012)*	(0.008)	(0.008)	
ln(similarity in country size)$_{it}$	1.728	0.069	0.581	
	(0.415)***	(0.294)	(0.254)**	
ln(relative physical capital endowment)$_{it}$	14.578	6.159	1.609	
	(5.368)***	(2.486)**	(1.493)	
ln(relative human capital endowment)$_{it}$	0.277	0.025	-0.054	
	(0.150)*	(0.138)	(0.133)	
ln(relative labor endowment)$_{it}$	-12.506	-5.802	-0.805	
	(5.379)**	(2.502)**	(1.352)	
Constant	-278.178	-32.544	-52.104	
	(46.223)***	(16.714)*	(31.223)*	
Year dummies	1990–1999	1991–1999	1991–1999	
Observations	2,767	2,198	2,198	
Units	341	337	337	
1st stage				
Instruments		56	56	
Arellano-Bond		z = -0.00	z = -0.01	
		(0.999)	(0.995)	
Hansen		$\chi^2_{37} = 36.79$	$\chi^2_{38} = 39.91$	
		(0.479)	(0.385)	
2nd stage				
Instruments			3	
Hansen			$\chi^2_1 = 0.59$	$\chi^2_1 = 2.94$
			(0.444)	(0.087)*
Short-run marginal effect of ln(distance)$_i$		0.891	0.939	
evaluated at the variable mean		(0.405)**	(0.567)*	(0.225)***
Long-run marginal effect of ln(distance)$_i$	15.274	5.380	8.259	
evaluated at the variable mean	(3.473)***	(2.698)**	(6.948)	(4.970)*

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Note: See Egger and Pfaffermayr (2004a) for a data description. We abbreviate the estimators as follows: "Within" denotes the least squares dummy variables estimator, and "HT-GLS" refers to the Hausman and Taylor (1981) generalized least squares estimator. "1s" and "2s" denote one-stage and two-stage estimators, respectively, and "sGMM" refers to system GMM estimators. We follow Blundell et al. (2000) to form the initial weighting matrix for feasible efficient estimation with "1s-sGMM" and the first stage of "2s-sGMM". We collapse the instrument matrices and for the equation in first differences we use the lags 2 to 4 of the dependent variable and the lags 0 to 4 of all other time-varying regressors as instruments. GMM standard errors are based on formula (9) with an unrestricted estimate of Ξ and the Windmeijer (2005) correction. Second-stage standard errors are based on formula (18). The standard errors are reported in parenthesis. All regressions include time dummies. The exogenous variables according to Assumption 2 are the similarity in country size and the relative human capital endowment. "Arellano-Bond" refers to the Arellano and Bond (1991) test for second-order serial correlation in the first-differenced residuals, and "Hansen" to the Hansen (1982) test of the overidentifying restrictions, with the p-values in parenthesis. The final column reports standard errors and test statistics for the "2s-sGMM" estimator based on uncorrected second-stage standard errors that do not take the first-stage estimation error into account. Coefficient estimates are unaffected.

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