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A structural model to study the bail-out process in a bank and its macro-prudential policy implications

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Abstract

In this paper, we construct a structural model to determine the costs of a bank rescue considering bail-outs and bail-ins. In our model, a government assumes the equity stake under unlimited liability upon abandonment of the original equity holders. The model determines an abandonment trigger such that if total income drops below this trigger, private shareholders abandon the bank. Given this trigger, the model also determines the bank rescue costs, the expected time to the bank rescue and the bank rescue probabilities. A static analysis of our model produces several empirically testable hypotheses. The model was explored in a sample of southern European countries considering alternative assumptions regarding parameter estimates and the behavior of operational costs. The model results regarding the rescue costs are reasonable, but the model also predicts bank rescues, estimates equity values, performs welfare analyses and estimates the impact of different macro- and micro-prudential policies. The empirical exercise we present, highlights the importance of the assumptions made regarding the behavior of the operational costs by showing dramatic differences in results in a sample of countries that otherwise appear to share important cultural and geographical proximities.

Keywords: structural model, abandonment trigger, bank bail-out, macro-prudential policies

JEL codes: G21, G28, H81

Non-Technical Summary

In this paper, we construct a structural model to determine the costs of a bank rescue considering bail-outs and bail-ins. In our structural model shareholders run a bank if the expected future cumulative income exceeds costs. When this net worth becomes negative, shareholders ultimately abandon the bank, forcing a governmental intervention.

In our model, a government assumes the equity stake upon abandonment of the original equity holders. Contrary to equity holders, which benefit from limited liability and are free to abandon operations when it is no longer profitable, the position of the government in our model is one of unlimited liability. In terms of the claims valued, in addition to the government and private shareholders, we also model the corporate debt value. To distinguish between bail-out and bail-in, we include a parameter ω, representing the percentage of corporate debt secured by the government in case of private equity holders' abandonment.

We perform a static analysis of our model, focusing mainly on the signs of the first partial derivatives. However, we complement this sign analysis with a numerical analysis given the richness of possible scenarios such as bail-out vs bail-in, high vs low volatility and the relative degree of efficiency of the private investors vs the government.

The empirical implications of the model were explored in a sample of southern European countries considering alternative assumptions regarding parameter estimates and the behavior of operational costs. The model results regarding the rescue costs are reasonable, but the model generates many more results such as predicting bank rescues, estimating equity values, performing welfare analyses and estimating the impact of different macro- and micro-prudential policies.

The empirical exercise we present highlights the main challenges of structural modeling, since our results are very sensitive to both the assumptions made regarding the behavior of the operational costs of the banks and to the origin of the data used for the estimation of the volatility and drift parameters. We have found dramatic differences in results for a sample of countries that otherwise appears to share important cultural and geographical proximities.

Since it is unlikely that the issue of bank failures will disappear from the public agenda in the near future, this model represents a tool to address the problems of bank rescues, but it is a simple first step.

1. Introduction

The subject of banking crises is currently high among public and research agendas in the wake of the recent financial crisis. Banking crises are associated with major micro- and macroeconomic adverse effects[1], and recent evidence indicates that no economy is immune to a banking crisis.

Following years of financial stability, there was a held belief that more-developed economies were safeguarded from banking crises. Unfortunately, recent evidence (see Reinhart and Rogoff, 2013) shows that the historical frequency of crises is similar for the developed and least-developed countries and that the negative effects of a banking crisis are large even when banking crises are non-systemic (Boyd, Kwak and Smith, 2005). With recent developments in mind, governments are now highly sensitive to banking crises and are ready to intervene even when there is no clear evidence of a market failure that requires government intervention[2].

Most of the current research being developed is still highly focused on analyzing the different costs incurred in a banking crisis (e.g., Reinhart and Rogoff, 2013) as well as the costs and benefits of the different policy instruments to address it (e.g., Philippon and Schnabl, 2009, and Veronesi and Zingales, 2010). However, we believe that more effort should also be devoted to predicting future problems and estimating the future costs of public bail-outs.

In this paper, we construct a structural model to determine the costs of a bail-out through which the government assumes the equity stake under unlimited liability, effectively becoming responsible for all the banks' financial obligations upon abandonment by the original private shareholders. Our cost measure—although limited in the sense that it does not take into account negative externalities such as the increase in public debt (see Reinhart and Rogoff, 2013) or the introduction of competitive distortions in the banking sector (see Hakenes and Schnabel, 2010 and Gropp, Hakenes and Schnabel, 2011)—represents the present value of the direct cost to avoid default and all future costs of maintaining the bank in operations. It also represents an equal and comparable measure of bail-out costs across different financial institutions, in which every financial institution is treated with the same remedy, avoiding the distorting results that may arise from the use of particular policy solutions made to fit the interests of influential and powerful incumbents (see Hart and Zingales, 2008, and Johnson, 2009).

Furthermore, it combines in a single cost measure all the different costs associated with the array of policy measures commonly used to address a banking crisis, such as equity injections, deposit insurance and loan guarantees. More importantly, our use of a structural model allows

[1] The literature shows us that banking crises are commonly associated with recessions and severe reductions in current and future GDP (Friedman and Schwartz, 1963 and Boyd, Kwak and Smith, 2005), stock market declines (Boyd, Kwak and Smith, 2005 and Dermine and Schoenmaker, 2010), currency crises (Kaminsky and Reinhart, 1999), overall loss of collateral values due to asset fire sales (Diamond and Dybvig, 1983), and contagion effects in the national financial institutions due to herding behavior (Calvo, 2005), or they are the result of bank runs or counterparty risks (Heider, Hoerova, and Holthausen, 2008), costs of financial distress (Korteweg, 2010) or underinvestment (Veronesi and Zingales, 2010).

[2] Clear evidence of the high sensitivity of governments to banking crises was the wave of deposit insurance level increases that took place in the third quarter of 2008, in which Ireland, Greece, Germany, Portugal and Denmark all announced that all bank deposits were insured at 100% within a period of weeks.

us to estimate the expected moment and probability of a bank rescue at the bank level. Our bank rescue approach through equity injections and the consideration of bail-outs and bail-ins fits well within our EU sample of banks used to explore the model. As Eschenbach and Schuknecht (2002) and Veronesi and Zingales (2010) note, equity financing may result in an increase in public debt but not an increase in the public deficit, something most EU countries are trying to avoid. Considering both bail-ins and bail-outs fits the current EU approach to bank rescues, as represented in Directive 2014/59/EU.

We apply our model to an array of EU banks determining the individual rescue costs, the likelihood of bank rescue, the equity estimates and a welfare value measure. We find our estimates of the bail-out costs to be reasonable when compared to actual rescue costs. The predictive nature of our model also allows the generation of warning signals and the implementation of macro- and micro-prudential measures aimed at avoiding future bank rescues. An example of these macro- and micro-prudential measures is presented in this paper.

The results are presented under a bail-out regime, which was the regime that applied when some of the banks in the sample were bailed out. However, we also estimate the expected bail-in costs following the spirit of Directive 2014/59/EU, by which all shareholders, creditors and depositors above a certain threshold will cover some portion of losses in case of a bank default.

Our paper differs from the existing literature in various aspects. In terms of the rescue costs, our approach is distinguished from others because it considers in a single policy action the objectives of alternative instruments such as loan guarantees, deposit insurance and equity injections. We also perform an ex ante estimation of the direct bail-out costs, whereas most papers that focus on determining bail-out costs perform ex post cost analyses (e.g., Kaminsky and Reinhart, 1999, Reinhart and Rogoff, 2013).

Our paper is also close to Veronesi and Zingales (2010) in that we also use a structural model to measure bank rescue costs. However, there are several important differences between our paper and Veronesi and Zingales (2010). First, in terms of the modeling approach, Veronesi and Zingales (2010) use a Merton-style approach, whereas we use a Leland (1994)-style model. We believe that the Leland (1994) approach is superior in that it allows for the realistic possibility of default driven by the inability to honor interest payments, whereas a Merton-style model only considers default at maturity of the debt claims[3]. Second, we consider a cash flow-based model and not an asset value approach, as in Veronesi and Zingales (2010). There are two main advantages to a cash flow-based model: (i) cash flows are an observable variable, whereas asset values are not, and (ii) cash flow-based models more-realistically incorporate changes in the

[3] Leland (1994) is commonly recognized as incorporating key contributions to the seminal work of Merton (1974). Notoriously, Leland is credited with the inclusion of tax shields and bankruptcy costs, effectively developing one of the first analytical trade-off models of capital structure (e.g., see Laajimi, 2012 and Sundaresan, 2013). Additionally, by considering endogenous default (first-passage model), Leland considers a richer set of default possibilities, including default triggered by the failure to make interest payments. Although our model includes taxes and bankruptcy costs, it was the endogenous default feature of Leland (1994) that led us to consider this modelling choice, to the detriment of Merton (1974).

corporate tax rates (e.g., see Laajimi, 2012)[4]. Finally, we combine in a single and simple cost measure all the different costs that Veronesi and Zingales (2010) have to estimate individually from equity injections, guarantees provided and co-insurance effects.

Our paper is also close to the literature on Early Warning Systems (EWSs) such as Bussiere and Fratzscher (2006) and Davis and Karim (2008) in the sense that we aim at predicting bank rescues, but again, there are several important differences. The main difference is that existing literature develops early warning systems for the financial system as a whole, whereas our model can be used in addition to an early warning system for each institution individually. Another important difference is that we use banks' fundamental values, such as the operating income or costs, whereas most of the existing literature only considers macroeconomic variables as EWSs or as functions of EWSs. Moreover, from a technical point of view, our approach is very different, since we consider a structural model, while the literature on EWS uses econometric models. The existing EWS literature uses methods that fall into two broad categories. The first approach (e.g., see Kaminsky et al., 1998, Kaminsky and Reinhart, 1999, or Goldstein et al., 2000) extracts signals from a range of macroeconomic indicators and considers banking crisis variables (e.g., aggregate capital, proportion of non-performing loans to total banking system assets). Then, by using signal extraction models, it detects whether variable aberrant behaviors can be quantitatively defined. The second approach uses binomial or multinomial logit or probit models (e.g., see Frankel and Rose, 1996, Berg and Pattillo, 1999, Bussiere and Fratzscher, 2006, or Davis and Karim, 2008). Under this second approach, which is like a scoring/rating system for the financial system as a whole, the probability of crisis for the whole financial system is calculated based on several factors (macroeconomic variables). The differences between our approach and classical EWS have very important and straightforward implications from a macro- and micro-prudential banking supervision viewpoint. Our structural model can be used as an early warning for the financial system as a whole and for individual institutions, since we focus on predicting bank failure and not a national bank crisis. Moreover, since our model is based on fundamentals, the early warning is, in most cases, based on concrete bank/banks data instead of global macroeconomic variables. Taking all the above into consideration, we believe that our approach of dealing with individual institutions is more appropriate for policymakers in implementing micro- and macro-prudential policies to prevent full-blown banking crises.

The structure of this paper is as follows: in section 2, we present our theoretical model for the bail-out and bail-in costs, discussing its main assumptions and discussing the results of various sensitivity analyses; in section 3, we present the data used to calibrate the model; section 4 presents the main results; finally, we conclude in section 5.

2. The model

In terms of decision-making, our economy comprises a government and banks, in which the government is committed to keeping banks operating even if private equity holders decide to

[4] An increase in corporate tax rates should reduce the value of the levered firm (e.g., see Goldstein et al., 2001), whereas in asset-based models such as Leland (1994), they increase it through the increase in the interest tax shields. To perform a realistic static analysis with an asset-based model requires that adjustments be made to the initial asset value to address this problem, such as those performed in Childs et al. (2005), something that cash flow models do not require.

abandon operations. Therefore, our model comprises two stages. In the initial stage, the banks are operated by wealth-maximizing private equity holders that are free to abandon the banking business when it is no longer profitable. In the second stage, following the abandonment of private equity holders, the banks are operated by a welfare-maximizing government committed to keeping the banks running and honoring its obligations towards its depositors. Before assuming control of a bank, the government assumes a passive role by merely enforcing its tax schedule and not taking any preemptive action in anticipation of a possible future bank rescue.

Contrary to equity holders, which benefit from limited liability and are free to abandon operations when it is no longer profitable, the position of the government in our model is one of unlimited liability. The governmental responsibility, in reality, takes many forms, such as deposit insurance, debt guarantees and equity injections, amongst many other forms of ensuring bank solvency. Such interventions reflect a going concern approach for the bank, eliminating uncertainty and tranquilizing creditors (depositors and other creditors such as bondholders), therefore preventing bank runs or the imposition of constraints by debtholders. In our model, the imposition of unlimited liability on the government, following the abandonment of the private equity holders, encapsulates in a single measure the effects of all the different types of governmental intervention aiming at reducing the risk of creditors. Naturally, unlimited liability will only be effective assuming symmetries of information, in which the different stakeholders rationally anticipate that upon abandonment of the private equity holders, the government steps in and assumes control of the bank, covering all cash flow shortfalls and fulfilling all of the banks' financial obligations. As such, our model assumes symmetries of information in which the government, private equity holders and the different creditors of the bank all have relevant information and rationally anticipate each other's actions. This assumption explains why, in this context, there is never the risk of a bank run or the emergence of underinvestment problems; however, the anticipation of a bail-out may expose the bank to moral hazard problems[5].

In terms of the claims valued, in addition to the government and private shareholders, we also model the corporate debt value. To distinguish between bail-out and bail-in, we include a parameter ω, representing the percentage of corporate debt secured by the government in case of private equity holders' abandonment. The bail-in case is very relevant nowadays, since Directive 2014/59/EU states that all equity holders, creditors and depositors (above a certain threshold[6]) are liable to assume some losses in the case of a bank default. Contrary to corporate debtholders, we do not directly consider the depositors' claims, because we assume that their position is secured in full through deposit insurance mechanisms.

When the bank rescue represents a pure bail-out (ω=1), the value of the corporate lenders is fixed and corporate debt is risk free. Naturally, in the bail-in case, ω<1 and ω have a lower limit

[5] We focus on the estimation of the strict bail-out costs for the government; however, our model can be easily extended to include investment decisions by the banks and to analyze the effect of these potentially important distortions.

[6] Although the directive establishes that depositors above a certain threshold may bear losses, for the sake of simplicity, we assume that all deposits are below this threshold, and consequently, the depositors' claim has no direct impact on our model. Naturally, the remuneration of deposits affects the decision of private shareholders to abandon the bank, an aspect that is taken into account in our model.

of zero, in which case, following the abandonment of private equity holders, the government does not assume any of the losses of corporate debtholders and strict absolute priority rules apply to determine the value of corporate debt.

In the following subsections, we detail the derivation of our bank rescue model.

2.1 Dynamics of the cash-flow process

Consider a bank that has uncertain operational cash-flow x represented by a Geometric Brownian motion (*gBm*),

$$dx = \mu_x x dt + \sigma_x x dz \quad (1)$$

where μ_x is the instantaneous growth rate of x, σ_x is the standard deviation, and dz is the increment of a standard Wiener process under the real-world measure $\mathbb{P}$. Naturally, $\mu_x < r$, with r being the return on a riskless asset, which allows us to obtain finite solutions[7]. Considering the existence of an appropriate exogenous factor Λ for discounting x, and following Thijssen (2010), we assume that Λ also follows *gBm*, as described in the following equation,

$$d\Lambda = r\Lambda dt + \sigma_\Lambda \Lambda dz \quad (2)$$

with the drift r representing the return on a riskless asset and σ_Λ, the constant standard deviation of Λ under the real-world measure $\mathbb{P}$, representing the exogenous price of cash flow risk. Assuming the existence of complete markets and no-arbitrage arguments, we use contingent claims analysis to value the different claims on x. As a result, under the risk-neutral measure $\mathbb{Q}$, the dynamics of x are described by

$$dx = (\mu_x - \sigma_x \sigma_\Lambda) x dt + \sigma_x x dz \quad (3)$$

in which the drift rate of x is reduced by the market risk premium $\sigma_x\sigma_\Lambda$, and dz now represents an increment of a standard Wiener process under the risk-neutral measure $\mathbb{Q}$. To simplify our notation, we define the return shortfall with uncertainty as δ, and $\delta = r + \sigma_x\sigma_\Lambda - \mu_x$.

2.2 Banking Costs

Regarding the cost structure of the bank, we consider two types of fixed costs. The first type is an operational cost that is defined as c_E when the bank is run by private equity holders and c_G when the bank is run by the government. This cost includes personnel, general and administrative expenses, depreciations and amortizations of long-term assets, write-offs and impairment charges and possible interest paid on bank deposits. Both c_E and c_G are assumed to be deterministic and fixed, but we consider that there may be a regime shift between c_E and c_G. The second type of costs we consider are purely financial costs, representing the interests paid on corporate debt; these are defined as i.

[7] This assumption may be justified in different ways. From a purely mathematical point of view, it represents a necessary condition to ensure the convergence of a perpetual geometric series. From the viewpoint of economics, it would be incongruent to assume that an asset could grow at a rate greater than the discount rate, because this would imply the creation of a bubble that would never burst. This assumption was probably first used in the seminal article by Gordon and Shapiro (1956), where, on pg. 106, the authors discuss its requirement.

When private equity holders abandon the bank and the government intervenes by rescuing the bank, we assume that the operational costs change from c_E to c_G. In our model, we generally assume that $c_E<c_G$ for the following reasons:

- Publicly operated firms are perceived to be less cost-efficient than privately operated firms (e.g., see Raff, 1992, Stroebel and van Benthem, 2010, Schwartz and Trolle, 2010, or Guriev et al, 2011). This lower efficiency is usually related to the transfer of resources to agents that provide political support to the government (see, for example, Shleifer, 1998, or Megginson, 2005) and to the fact that governments are not, by nature, profit-oriented (see Stiglitz, 2000).
- The cost structure of banks in troubled times differs from that in normal times. In this case, there is a natural expectation of increases in write-offs and impairment charges.

In our model, we consider that the change in regime occurs when the equity holders abandon the bank and the government steps in to rescue the bank. However, in reality, the regime change from c_E to c_G may take place while the bank is still run by the private equity holders. We have several cases in which the regime shift of increases in write-offs and impairment charges occurs while private shareholders are running the bank. The regime shift may occur following managerial changes (the new managerial team wishes to start with a clean slate) or in anticipation of an imminent public rescue with the aim to secure more funds for the rescue package of the bank. Reality presents us with a variety of possible and reasonable scenarios. Therefore, although in the theoretical section we explore a strict fixed cost nature with a regime shift, in the empirical sections, we consider alternative assumptions regarding the private and public costs and the fixed and variable nature of the operational costs.

2.3 Valuing the different claims

In our model, we explicitly value four different claims: the private equity holders, defined as $E(x)$; the government, defined as $G(x)$; corporate debtholders, defined as $D(x)$; and a fourth claim, defined as $R(x)$, which represents the rescue cost for a government by assuming control of a bank following the abandonment of private equity holders. It is important to clarify that the sum of $E(x)$ and $D(x)$ does not represent the value of the bank, since there are also bank deposits financing the assets of the bank. However, we do not value the claim of bank depositors explicitly, because, as stated previously, we assume that regardless of the type of rescue implemented (e.g., bail-out, bail-in), depositors are fully protected, and therefore, their value remains constant over time.

The value dynamics for the claims we consider are very similar, differing only in what concerns the individual cash-flow streams (π) and the abandonment values (L). Therefore, and for the sake of brevity and clarity, we describe the derivation process for a general claim A in which A = E, G, D, R for the cases of the private equity holders, government, corporate debtholders and rescue costs, respectively. Naturally, the important differences between the claims are thoroughly discussed as they arise.

The following Ordinary Differential Equation (*ODE*) describes the value of the general claim A.

$$0.5\sigma_x^2 x^2 \frac{\partial^2 A}{\partial x^2} + (\mu_x - \sigma_x \sigma_\Lambda) x \frac{\partial A}{\partial x} - rA + \pi = 0 \tag{4}$$

where π represents the cash flow accruing to each claim, comprising a variable component a associated with the evolution of x and a fixed component b independent of x, and naturally, $\pi = ax+b$. The cash flows are subject to corporate taxation, and it is assumed that the government runs a symmetrical corporate tax scheme to which it is committed and in which firms are taxed at a rate defined as τ. For the different claims considered, Table 1 defines a, b and π. The terminal values L are presented and discussed with the terminal value-matching boundary conditions.

Table 1: Specification of the ODE

$A(x)$	a	B	π
$E(x)$	$1 - \tau$	$-(c_E + i)(1 - \tau)$	$(x - c_E + i)(1 - \tau)$
$G(x)$	T	$-(c_E + i)\tau$	$(x - c_E + i)\tau$
$D(x)$	0	I	i
$B(x)$	1	$-c_G$	$x - c_G$

Notes: This table defines a, b and π for the different claims considered.

We assume a residual dividend policy both when the bank is operated by private equity holders and when the government operates the bank. When the bank is operated by private equity holders and as long as $x>c_E+i$, the dividend is equal to $(x - c_E - i)(1 - \tau)$. Whenever $x<c_E+i$, equity holders make the necessary cash injections to avoid default (they basically recapitalize the bank with equity injections as the need arises). Equity holders are willing to capitalize the bank through regular cash injections for as long as these cash injections do not exceed the market value of their shares. The government receives a flow of corporate taxes equal to $(x - c_E - i)\tau$, and, similarly to equity holders, this flow may be negative when $x<c_E+i$. In this case, and given the symmetrical nature of the tax system, the negative flows proxy the governmental return of past taxes paid or the negative earnings that are carried forward. While private equity holders run the bank, a stream of continuous interest payments i is made to corporate lenders.

Following the abandonment of the private equity holders, the government steps in assuming the role of an equity holder, rescuing the bank and participating in its management; but when assuming control, the government also restructures its corporate debt. Therefore, previous equity holders face a terminal value of zero enforcing limited liability and absolute priority rules, and the government will pay no more interest to corporate lenders (i). Following the rescue, the flow of earnings received by the government simplifies to $(x - c_G)$. At this stage, the government assumes a double role of receiving dividends and corporate taxes. The dividends are represented by $(x - c_G)(1 - \tau)$, and the corporate taxes are represented by $(x - c_G)\tau$. When we add the two flows, the corporate taxes paid and received cancel out and the net flow for the government simplifies to $x - c_G$.

The general solution to ODE (4) is:

$$A(x)=\begin{cases}\dfrac{ax}{\delta}+\dfrac{b}{r}+B_1x^{\beta_1}+B_2x^{\beta 2} & \text{if } x>x_L\\ L_A & \text{if } x\le x_L\end{cases} \tag{5}$$

In which a and b are given in Table 1, B_1 and B_2 are constants to be determined given appropriate boundary conditions and β_1 and β_2 are the roots to the following characteristic polynomial:

$$(0.5\sigma_x^2\beta^2+\beta(\mu_x-\sigma_x\sigma_\Lambda-0.5\sigma_x^2)-r)x^\beta=0 \tag{6}$$

yielding

$$\beta_1=0.5-\frac{\mu_x-\sigma_x\sigma_\Lambda}{\sigma_x^2}+\sqrt{\left(0.5-\frac{\mu_x-\sigma_x\sigma_\Lambda}{\sigma_x^2}\right)^2+\frac{2r}{\sigma_x^2}}>1 \tag{7}$$

$$\beta_2=0.5-\frac{\mu_x-\sigma_x\sigma_\Lambda}{\sigma_x^2}-\sqrt{\left(0.5-\frac{\mu_x-\sigma_x\sigma_\Lambda}{\sigma_x^2}\right)^2+\frac{2r}{\sigma_x^2}}<0 \tag{8}$$

Because $x\rightarrow\infty$, the likelihood of a bank rescue becomes negligible, as stated in condition (9),

$$\lim_{x\to\infty}A(x)=\frac{ax}{\delta}+\frac{b}{r} \tag{9}$$

implying that for all the claims considered, the constant B_1 will be equal to zero. The constant B_2 is determined by the following abandonment value-matching condition (x=x_L),

$$A(x_L)=L_A \tag{10}$$

Replacing in (5) B_1 with 0 and replacing B_2 with its solution obtained from boundary (10), we obtain the particular solution for the value of a general claim $A(x)$,

$$A(x)=\begin{cases}\dfrac{ax}{\delta}+\dfrac{b}{r}-\left(\dfrac{ax_L}{\delta}+\dfrac{b}{r}-L_A\right)\left(\dfrac{x}{x_L}\right)^{\beta_2} & \text{if } x>x_L\\ L_A & \text{if } x\le x_L\end{cases} \tag{11}$$

The value matching boundary conditions are crucial to our model, since they define the nature of the bank rescue and are important to understand the intuition behind our model. Therefore, we present and discuss these value-matching conditions next, from where L_A is determined.

$$E(x_L)=0 \tag{12}$$

$$G(x_L)=\frac{x_L}{\delta}-\frac{c_G}{r}-Min\left\{\frac{i}{r};Max\left\{\frac{i\omega}{r};\frac{x_L}{\delta}-\frac{c_G}{r}\right\}\right\} \tag{13}$$

$$D(x_L)=Min\left\{\frac{i}{r};Max\left\{\frac{x_L}{\delta}-\frac{c_G}{r};\frac{i\omega}{r}\right\}\right\} \tag{14}$$

Starting with equity, value-matching boundary (12) enforces absolute priority and limited liability rules stating that upon abandonment, equity holders receive nothing from the bank, but they are also not responsible for any contractual obligations that cannot be satisfied.

To help understand boundaries (13) and (14), we briefly analyze the following two value components:

$$\frac{x_L}{\delta} - \frac{c_G}{r}, \tag{15}$$

$$\frac{i}{r}. \tag{16}$$

Expression (15) represents the pure operational value of the bank absent taxes and following a financial restructuring (corporate debt will be partially or fully repaid and the bank is now unlevered), because the government has already rescued the bank. This operational value may be positive or negative.

Expression (16) represents the present value of corporate lenders when the bank honors all its obligations towards them by paying the full stream of coupons.

Regarding the government, boundary (13) states that following the abandonment of the original equity holders, the government assumes control and financially restructures the bank, eliminating financial leverage. Thus, in the general case, the value for the government represents the difference between the operational value of the bank and the value repaid to corporate lenders [(15)-(16)]. Strictly speaking, this is the case of a full bail-out. In the case of a bail-in, the Government only pays creditors a fraction ω of what they are contractually owed. However, there are two alternative possibilities that justify the use of the max and min operators in boundaries (13)-(14):

- For a bail-in, the operational value of the bank may be larger than the value payable to corporate lenders [(15)>ω(16)] but smaller than (16). In this case, the government is forced to respect absolute priority rules, and the lender is entitled to the positive operational value of the bank (given by expression (15)), and the value for the government is 0.
- For both cases of bail-out and bail-in, the operational value of the bank may be larger than (16). In this case, the lenders recover their investment in full (expression 16), and the positive difference (15) – (16) accrues to the government.

Two important aspects of boundary (13) deserve a brief discussion:

- First, note that boundary (13) does not include any option value. This fact reflects the unlimited liability role assumed by the government when it assumes control of the failed bank. It states that once the bank is rescued, the government bears all the costs associated with the terms of the bail-out or bail-in. Boundary (13) therefore represents the true measure of the rescue costs for the government[8].

[8] The bank rescue is associated with losses for as long as $c_G>c_E$. The government would be able to generate gains with the rescue only in the unrealistic case of having very inefficient private equity holders when compared to the government. Naturally, this case is of no concern to policymakers.

- Second, boundary (13) states that the government assumes perpetual ownership of the bank, which appears unreasonable when, in reality, following its restructuring (either operational or financial), governments usually sell the bank back to private investors. This possibility is not taken into account, because it would have no impact on the value (cost) of the bank rescue for the government. It is true that following the government restructuring of the bank, it is likely that private investors will be willing to purchase it back. However, it is also true that the selling price of the restructured bank will never exceed the restructuring costs. If the bank restructuring represented a positive net present value project, it is obvious that the private equity holders would have proceeded with the restructuring of the bank instead of its abandonment. Therefore, even if we take into account the possibility of re-selling the bank back into private hands, the net rescue costs for the government are still accurately measured by boundary (13).

Having developed the appropriate value-matching boundary conditions at the abandonment of private equity holders, we are now able to obtain the value of all individual claims. From equation (11), if we replace *a* and *b* with the appropriate expressions from Table 1 and replace L_A with the appropriate value-matching abandonment condition values, we obtain all the pertinent value equations. The following Proposition 1 presents the equations for the values of Equity holders, the Government and Corporate lenders before the bank rescue.

Proposition 1. *The value accruing to Equity holders* ($E(x)$), *Debtholders* ($D(x)$) *and the Government* ($G(x)$) *of a bank run by private equity holders is given by*

$$E(x)=\begin{cases}\left(\frac{x}{\delta}-\frac{c_E+i}{r}\right)(1-\tau)-\left(\frac{x_L}{\delta}-\frac{c_E+i}{r}\right)(1-\tau)\left(\frac{x}{x_L}\right)^{\beta_2} & \text{if } x>x_L\\ 0 & \text{if } x\le x_L\end{cases}, \tag{17}$$

$$G(x)=\begin{cases}\left(\frac{x}{\delta}-\frac{c_E+i}{r}\right)\tau-\left[\frac{x_L(\tau-1)}{\delta}-\frac{(c_E+i)\tau-c_G}{r}+Min\left\{\frac{i}{r};Max\left\{\frac{i\omega}{r};\frac{x_L}{\delta}-\frac{c_G}{r}\right\}\right\}\right]\left(\frac{x}{x_L}\right)^{\beta_2} & \text{if } x>x_L\\ \frac{x_L}{\delta}-\frac{c_G}{r}-Min\left\{\frac{i}{r};Max\left\{\frac{i\omega}{r};\frac{x_L}{\delta}-\frac{c_G}{r}\right\}\right\} & \text{if } x\le x_L\end{cases}$$

, (18)

$$D(x)=\begin{cases}\frac{i}{r}-\left(\frac{i}{r}-Min\left\{\frac{i}{r};Max\left\{\frac{x_L}{\delta}-\frac{c_G}{r};\frac{i\omega}{r}\right\}\right\}\right)\left(\frac{x}{x_L}\right)^{\beta_2} & \text{if } x>x_L\\ Min\left\{\frac{i}{r};Max\left\{\frac{x_L}{\delta}-\frac{c_G}{r};\frac{i\omega}{r}\right\}\right\} & \text{if } x\le x_L\end{cases}, \tag{19}$$

in which ω *represents the bail-out/bail-in parameter* (ω=1 *stands for a full bail-out and* ω<1 *stands for a bail-in),* $\delta = r + \sigma_x\sigma_\Lambda - \mu_x$ *represents the return shortfall and* x_L *represents the abandonment trigger of equity holders.*

The first term of equation (17) represents the present value of the perpetual stream of dividends to equity holders. However, as stated previously, the dividends may be negative (for $x<c_E+i$) and effectively become equity injections. By enforcing limited liability, equity holders may choose to

abandon the bank, and the present value of this abandonment option is what the second term of equation (17) represents.

While the private equity holders run the bank, the role of the government is merely to collect taxes when $x>c_E+i$ and to 'return' taxes when $x<c_E+i$. This is what the first term of equation (18) reflects for $x>x_L$. However, following the abandonment of equity holders (when $x<x_L$), the government assumes a double role as equity holder and tax collector. Furthermore, the government financially restructures the bank. The second term of equation (18), for $x>x_L$, reflects the change in the role of the government from a strict tax collector to its double role as tax collector and equity holder. Furthermore, it takes into account the outcome of the financial restructuring of the bank. In equation (18), when $x=x_L$, the government operates an unlevered bank, but it must pay corporate lenders the maximum between what they are entitled to following absolute priority rules and the terms set for the rescue (either bail-out or bail-in). In fact, this term sums up the net position for the government of rescuing the bank.

Finally, equation (19) for $x>x_L$ represents the value for corporate lenders when the bank is still operated by private equity holders. The first term reflects the present value of a perpetual string of coupons. The second term incorporates the probability of abandonment by equity holders. It also incorporates the loss of coupons and the capital recovery. The capital recovery represents the maximum between the fair value for lenders if absolute priority rules are respected, as well as the terms imposed by the bail-out or bail-in.

2.4 Bank rescue

Having all the value equations, we need to determine the moment of abandonment of the private equity holders, or, in other words, the expected moment at which the bank will require governmental rescue.

Since the bank rescue is triggered with the abandonment of the private equity holders, we have to determine when the private equity holders optimally abandon operations of the bank. The abandonment trigger (x_L) is obtained from the following smooth pasting or optimality condition.

$$\left.\frac{\partial E(x)}{\partial x}\right|_{x=x_L}=0 \tag{20}$$

Abandonment is then expressed as a function of x; however, it is more intuitive to express it as a function of expected time to abandonment or in terms of conditional probabilities. Therefore, we will to convert x_L into an expected time to exercise and into a probability of occurrence within a given time frame ($t_0\leq t\leq T$).

The following proposition summarizes the rescue trigger of the bank, the expected time to rescue, the probability of a bank rescue taking place and the bank rescue costs for the government.

Proposition 2. *The moment at which the bank rescue takes place expressed in terms of earnings level (x_L), expected time (ϑ_L), and probability of occurrence within a given time frame ($P(x \leq x_L)$) and the value of the governmental rescue at the moment it occurs are given by*

$$x_L = \frac{\beta_2}{\beta_2 - 1} \frac{\delta(c_E + i)}{r}, \tag{21}$$

$$\theta_L = \frac{1}{\mu_x - 0.5\sigma_x^2} \ln\left(\frac{x_L}{x}\right) \tag{22}$$

$$P(x < x_L \mid t_0 \leq t \leq T) = 1 + \Phi\left(\frac{\ln\left(\frac{x_L}{x}\right) + \left(\mu_x - \frac{\sigma_x^{\,2}}{2}\right)T}{\sigma_x \sqrt{T}}\right) e^{\frac{2\left(\mu_x - \frac{\sigma_x^{\,2}}{2}\right)\ln\left(\frac{x_L}{x}\right)}{\sigma_x^{\,2}}} - \Phi\left(-\frac{\ln\left(\frac{x_L}{x}\right) - \left(\mu_x - \frac{\sigma_x^{\,2}}{2}\right)T}{\sigma_x \sqrt{T}}\right) \tag{23}$$

$$R(x_L) = \frac{x_L}{\delta} - \frac{c_G}{r} - Min\left\{\frac{i}{r}; Max\left\{\frac{i\omega}{r}; \frac{x_L}{\delta} - \frac{c_G}{r}\right\}\right\}, \tag{24}$$

in which ω represents the bail-out/bail-in parameter (ω=1 stands for a full bail-out and ω<1 stands for a bail-in), $\delta = r + \sigma_x \sigma_\Lambda - \mu_x$ represents the return shortfall and x_L represents the abandonment trigger of equity holders.

In the following section, we perform a static analysis of our bank rescue model.

2.5 Sensitivity analysis

In this section, we perform a static analysis of our model, focusing mainly on the signs of the first partial derivatives. However, we complement this sign analysis with a numerical analysis given the richness of possible scenarios such as bail-out vs bail-in, high vs low volatility and the relative degree of efficiency of the private investors vs the government. The use of the numerical analysis is entirely justified, since in this type of model, the sensitivities of the results to changes in the base parameters can be quite volatile (e.g., see Leland, 1994).

Table 2 presents the sensitivity analysis results for the general case of a bail-in, when x has moderate volatility and private investors are more efficient than the government. The particular cases are discussed in the body of the text.

Table 2: Sensitivity analysis

	x_L	ϑ_L	$P(x<x_L)$	$R(x_L)$	$E(x)$	$D(x)$	$G(x)$
μ_x	↓	↑	↓	↑	↑	↑	↑
σ_x	↓	↓	↑	↓	↓	↓	↓
σ_Λ	↑	↓	↑	↓	↓	↓	↓
R	↑	↓	↑	↑	↓	↓	↓
T	-	-	-	-	↓	-	↑
Ω	-	-	-	↓	-	↑	↓
c_E	↑	↓	↑	↑	↓	↓	↓
c_G	-	-	-	↓	-	↓	↓
I	↑	↓	↑	↓	↓	↑	↑

Notes: This table reports how the bank rescue threshold (xL), the expected time to rescue (θL), the rescue probability (P), the bank rescue cost (R) and the value of the different claims (E, D and G) change in response to increases in the parameter values (μx, σx, σΛ, r, τ, ω, cE, cG and i).

2.5.1 Sensitivity of claim values

There are two periods in the life of all the claims we considered: the period of normal functioning while the bank is operated by the private equity holders and the period following the bank rescue, when the bank becomes operated by the government. The abandonment trigger of equity holders x_L marks the change between the two periods and defines the expected length of the normal period during which the bank does not require public assistance. During this period, equity holders, debtholders and the government receive dividends, interest and taxes, respectively. In general, as the length of this normal period increases (every time a change in a parameter value decreases x_L) the values of the equity holders, debtholders and the governmental claim increase. This becomes clear with the analysis of the impact of changes in μ_x, r and c_E. For all these cases, increases (decreases) in x_L are generally associated with a decrease (increase) in the values of $E(x)$, $D(x)$ and $G(x)$. In the particular case of gross inefficiency of the private equity holders vs the government (when $c_E >> c_G$), both $D(x)$ and $G(x)$ increase with the abandonment of equity holders due to the fact that given their inefficiency, the operational value of the bank increases following abandonment, benefiting the debtholders and the government alike.

Changes in the governmental costs do not affect $E(x)$, due to absolute priority rules. For the cases of D(x) and $G(x)$, increases in c_G are generally associated with a reduction in the value of both claims. The bail-in / bail-out parameter ω includes a transfer of wealth from the government to debtholders by partially (bail-in) or totally (bail-out) securing the value of the debt claims. In this case, increases in ω imply an increase in $D(x)$ and a corresponding decrease in $G(x)$.

Some parameters are directly associated with individual claims, such as i and τ, which are linked to the income of debtholders and of the government, respectively. As such, increases in these parameters increase the value of the individual claims $D(x)$ and $G(x)$ at the expense of the other claims. Naturally, there are some exceptions; debtholders are indifferent to changes in corporate taxes, because coupons have priority over corporate taxes[9]. In the case of a bail-out, $D(x)$ is also insensitive to any change in the parameter values, because the government will bail out debtholders. In the case of a bail-in, the effects of changes in i may be the opposite of what Table 2 reports. Whenever the bank is close to being abandoned ($x \rightarrow x_L$), debtholders benefit from a decrease in i, since this decrease induces equity holders to delay default, allowing debtholders to collect interest for a longer period (this effect is well documented and was first pointed out in Stiglitz and Weiss, 1981).

With respect to the volatility parameters σ_x and σ_Λ, the effects are not as linear as with the previous parameters. Both are volatility parameters and both affect the drift component of x and, consequently, the discount imposed on x. However, only σ_x is directly associated with the diffusion of x, and therefore, it affects the speed of changes to x. The drift effect for increases in either volatility source translates into a decrease in the value of the operating bank and, therefore, decreases in $E(x)$, $D(x)$ and $G(x)$. However, since increases in σ_x affect the speed of changes in x, contrary to increases in σ_Λ that accelerate abandonment, increases in σ_x actually delay abandonment[10]. For high values of σ_x, $E(x)$ actually increases because the value of the option begins to dominate the value of the operating bank. Naturally, the claims $D(x)$ and $G(x)$ do not benefit from the increase in the optionality value, since the abandonment option is owned by equity holders. Several second-order effects occur from the interactions between both volatility parameters. However, although they are theoretically possible, they are unimportant for our model. Therefore, we refer the reader to Thijssen (2010) for a discussion of these effects.

2.5.2 Rescue timing and probabilities

The timing of the rescue is determined by the unwillingness of equity holders to continue capitalizing the bank. As such, the timing of the bank rescue is determined by the abandonment trigger of equity holders x_L. Whenever a change in a parameter value increases x_L, the rescue occurs earlier than expected. Therefore, with an increase in x_L, we have decreases in the expected time to the rescue (ϑ_L) and an increase in the bank rescue probability (P). This is the pattern we observe in the cases of changes in μ_x, σ_Λ, r, c_E and i. However, there is an exception to this pattern associated with changes in σ_x. Increases in σ_x translate into increases in the value of the abandonment option and, consequently, in a reduction in x_L. Nonetheless, although x_L is lower, the increase in volatility makes it more likely that the abandonment trigger will be reached, and this is reflected in the decreases we observe in ϑ_L and P for increases in σ_x (the effect described in footnote 6).

The willingness of equity holders to abandon the bank increases when the present value of their shares decreases due to an increase in costs or a reduction in dividends. The banking costs include two components, an operational component c_E and a financial component i. Thus, increases in either c_E or i accelerate abandonment and the consequent governmental rescue of

[9] Remember that our model assumes a residual dividend policy; if this were not the case, the relation could be negative, although relatively insignificant.

[10] When we state that abandonment is delayed, we refer to the decrease in x_L. However, even though the abandonment trigger is lower, it is much more likely that it will be reached. Therefore, the expected time to abandonment (ϑ) and the probability of abandonment (P) both increase.

the bank. Regarding taxes, the symmetrical nature of the tax system makes the abandonment decision neutral to changes in the corporate tax rate; therefore, changes in τ do not affect the decision of equity holders to abandon. The present value of the dividends decreases with decreases in μ_x, increases in r and increases in σ_Λ. In all these cases, equity holders accelerate abandonment. The case of σ_x was previously discussed; in this case, although increases in σ_x induce a reduction in x_L, the likelihood of abandonment is actually increased, since a higher volatility of x makes it more likely that the x_L trigger will be reached.

2.5.3 Rescue costs

The sensitivity of rescue costs to changes in the parameter values is best understood by considering the fact that the rescue costs reflect the value of bank ownership following the abandonment of private equity holders[11]. We assume that ownership of the bank is associated with the cost of having to pay debtholders; however, there is also the value generated by the assets of the bank.

Therefore, the rescue costs increase with increases in ω and i whenever the terms of the bail-in are generous to debtholders. Paradoxically, when the bail-in imposes important losses on debtholders, we observe that the rescue costs actually decrease with increases in i. This effect is explained by the fact that equity holders abandon earlier when i increases, and therefore, the government assumes control of a financially restructured bank at higher values of x while at the same time bearing relatively small costs for the financial restructuring by imposing harsh bail-in conditions on its debtholders. Regarding taxes, following the bank rescue, the government assumes a double role as a taxpayer and a tax collector, so the rescue costs are insensitive to changes in the corporate taxes.

Since the rescue costs derive from an ownership position, there are some similarities between the sensitivities of $R(x_L)$ and $E(x)$, such as increases in both claims when the value of the bank is positively affected by increases in μ_x and decreases in the operational costs c[12]. The government assumes ownership of the bank following the abandonment of private equity holders, so even taking into account that the bank is financially restructured, the cash flows generated are most likely negative. Therefore, increases in r reduce the rescue costs.

Regarding the volatility parameters, increases in σ_x and σ_Λ translate into increases in the rescue costs by accelerating the bank rescue and by reducing the present value of the bank due to the decrease in the drift of x.

2.5.4 Bail-out vs bail-in when the cost of debt is affected

In the sensitivity analysis discussed previously, we observed how increases in ω imply a decrease in the rescue costs. In simple words, the government will save money when it moves from a

[11] It is important to remember that contrary to $E(x)$, which by construction is always positive or zero, given limited liability, $R(x_L)$ will be negative in expectation. Therefore, increases in $R(x_L)$ reflect a decrease in the rescue costs, and decreases in $R(x_L)$ represent an increase in the rescue costs.

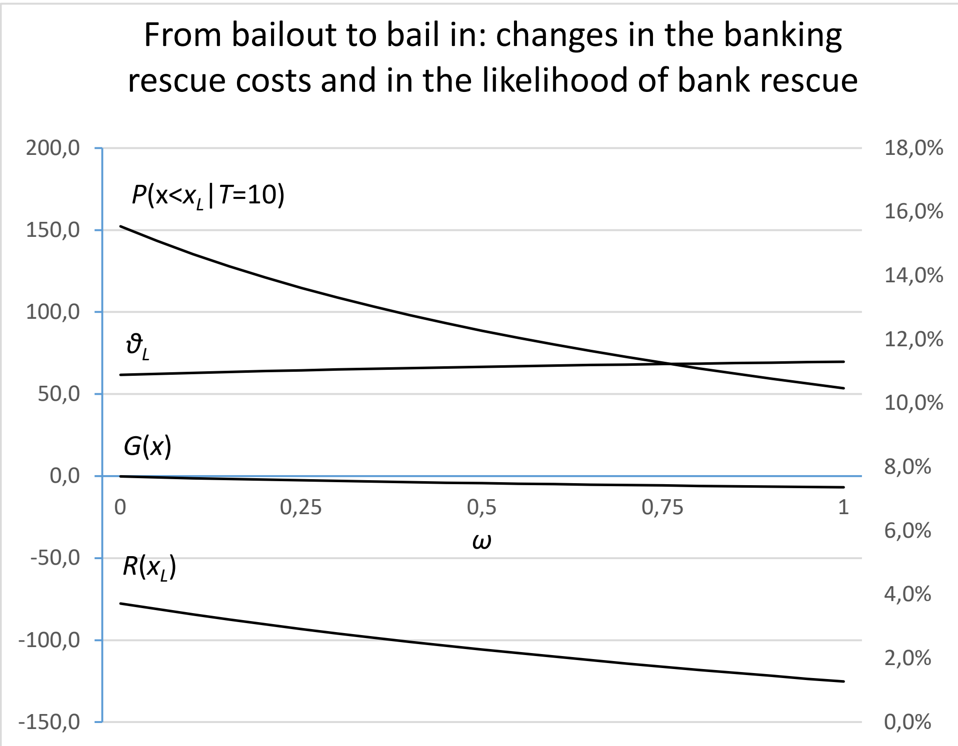
[12] Although there are similarities, there are also fundamental differences between claims $E(x)$ and $R(x_L)$. Most importantly, $E(x)$ includes an optionality feature given by limited liability. This feature is absent from $R(x_L)$, making it essentially an unlimited liability claim.

policy of bailing-out corporate lenders to a bail-in policy while imposing some costs on the lenders by transforming risk-free debt into risky debt.

Our previous analysis only considered the impact of changes in ω on existing debt, thus ignoring potentially important interactions between variables ω and i.

Absent asymmetries of information and assuming the rationality of debtholders, when the government changes from a bail-out to a bail-in policy, in which the terms of the rescue are public, new corporate debt will naturally be issued at higher costs as a response to the increase in credit risk (as ω decreases, i increases as a response). The increase in the financing costs will naturally translate into an accelerated abandonment (i.e., increase in x_L), increasing the likelihood of a bank rescue (i.e., decrease in ϑ_L and increase in P) with an uncertain impact on the rescue costs. These effects lead us to question whether a bail-in is in fact cheaper than a bail-out. To answer this question, we perform a simple analysis in which changes in ω are associated with increases in i so that the value of the debtholders' claim remains unchanged as ω varies from 0 to 1.

Figure 1: Bail-out vs bail-in costs



Notes: This plot presents the effects on P, ϑ_L, $R(x_L)$ and $G(x)$ considering simultaneous changes in ω and i. The primary vertical axis is associated with the variables ϑ_L, $R(x_L)$ and $G(x)$, and the secondary vertical axis is associated with variable P.

As is clear from the analysis of Figure 1, the bank rescue costs increase with increases in ω (both $R(x_L)$ and $G(x)$ decrease as ω increases). In simple words, by imposing some costs on corporate lenders, the government in fact reduces the bank rescue costs, even considering the feedback effect of corporate lenders increasing the cost of new debt as a response to the decrease in capital recovery. However, when we take into account the response of corporate lenders to the change in rescue policy, we also observe that a bail-in policy accelerates the bank rescue and consequently increases the rescue probability[13].

In summary, our results show that the move from a bail-out to a bail-in approach dictated by Directive 2014/59/EU will most likely result in an increase in the need for bank rescues, although with lower rescue costs for the governments.

3. Data sample and calibrations

Our model is a cash flow-based model; however, it is not possible to obtain direct estimates for the financial flows of the banks. Therefore, we make the standard assumption of proxying the financial flows with economic (accounting) flows. As a result, we work with operating income and expense figures, including asset-write downs, to take into account the negative cash flows associated with unrecoverable capital[14]. Our dataset comprises quarterly figures from the SNL Financial Database covering the period from the first quarter of 2008 to the third quarter of 2014 for 41 individual banks from Cyprus (CY), Spain (ES), Greece (GR), Italy (IT) and Portugal (PT)[15]. We work with the SNL figures for operating income (SNL KeyField: 225155), net interest income (SNL KeyField:132553), operating expenses (SNL KeyField: 248802) and asset write-downs (SNL KeyField: 225181). As stated below, in this paper, operating income will be used as a proxy of total income, and the sum of operating expenses and asset write-downs will be used as a proxy of total cost. Moreover, in section 4.4 (Macroprudential Policies), we will simulate bank results assuming that the banks' total income becomes simply net interest income, and cost will be adjusted accordingly.

Working with financial statements with quarterly frequencies implies that we will have to address the problem of missing data points. Thus, we start with a subsample of banks with complete datasets that we use to construct an interim country sub-aggregate. We estimate the growth rates from these sub-aggregates and use them to interpolate and extrapolate missing data points for the individual banks dataset. Thus, all the estimates presented in this paper are calculated from this completed dataset.

With respect to the cost of debt, since our model considers a console bond, for each bank, we estimate the parameter *i* (the console coupon) as the product of the book value of financial debt and the risk-free rate. The book value of total debt is obtained from the SNL database (SNL KeyField: 132319) and considers the principal balance of all the financial obligations of the bank. The risk-free rate considered is 6%, in line with the average Euro Area Long Term Bond Yields for the period 1997-2007, and the volatility of the discount factor (σ_Λ) is assumed to be 0.2[16]. The

[13] We performed a similar analysis with various combinations of parameter values, and the qualitative nature of the results held constant.

[14] Consider the case of a Non-Performing Loan. The loss of interest is captured by the reduction in income (our x variable), while the debt principal lost is captured by the asset write-down.

[15] We have chosen these countries because they represent a homogenous sample of the banking systems that are more affected by the recent financial crisis.

[16] There is wide variability in the literature regarding σ_Λ, where we can find figures as low as 0.0468 and 0.1 in Huang and Huang (2012) and Thijssen (2006), respectively, and as high as 0.3 and 0.4 in Thijssen (2010) and Glover and Hambusch (2013), respectively. Although we consider a mid-term of the various figures previously used, we also analyzed the sensitivity of our main results to changes of σ_Λ from 0.1 to 0.3 and found that these changes do not significantly affect our results.

corporate tax rate assumed is 25%, a figure in line with the GDP weighted European average of 26.22% according to the Tax Foundation (2016).

In our empirical analysis, we take a more flexible approach regarding the operational costs with respect to the theoretical model presented and analyzed in section 2. In our theoretical analysis of a bank rescue, we assumed that the operational costs are fixed and that there is a level change whenever the government assumes control of the bank. In our empirical analysis, this will be our Scenario #1. However, we consider additional scenarios. In Scenario #2, we still assume the level change in costs following the bank rescue, but we assume that the operational costs are now variable and we discount them not at the risk-free rate but at the parameter δ. In Scenario #3, we disregard the level change in the operational costs but consider that the operational costs are fixed in nature. Finally, in Scenario #4, we disregard the level change in the operational costs and, similar to Scenario #2, consider that the operational costs are variable in nature.

The details on the estimations of the drift and risk parameters are given in section 3.1. The details of the operational cost parameters for the government and the private equity holders are given in section 3.2.

3.1 Estimating the drift and risk parameters

The drift and risk parameters are crucial for the valuation process, more specifically concerning the capitalizations and discounting procedures. Since we are working with a relatively short data series, to obtain reliable figures for the μ_x and σ_x parameters, we perform two estimations. The first estimation is performed at the country level, and the second estimation is performed at the bank level. Because total income x, which in this paper is represented by operating income (an accounting value, as stated above), follows *gBm*, the distribution of its logarithm differences is Gaussian.

Bayesian inference about the parameters herein follows the exposition in Richardson (2011) adapted from Hoff (2009), see Appendix 2, whereby one assumes independent prior distributions for the mean and precision (reciprocal of variance) of this Normal distribution, a Normal and a Gamma, respectively. Because of this independence assumption, prior beliefs about mean and precision may arise from different sources. Using a Gibbs sampler, we obtain a dependent sequence of posterior means and precisions. The details of how we initially introduce our beliefs about the drift μ_x and risk σ_x parameters into the prior distributions are presented in the Appendix 2.

In the country-level estimation, we arbitrarily introduce the initial beliefs μ_x = 0% and σ_x = 20% and estimate both parameters using country-level data[17]. In the bank-level estimation, we use

[17] The importance of the domestic market is quite significant within our sample of banks. Therefore, it is reasonable to assume that within a single country, the dynamics of the market for banking services are similar for every bank. Thus, for each country, we aggregate the total income of the individual banks and use these aggregate figures to estimate country growth rates (the μ_x parameter) and their standard deviation (the σ_x parameter). This procedure also reduces the impact of important outliers observed in our parameter estimations considering individual quarterly bank data. Different accounting practices when producing the financial statements of the individual banks are possibly at the origin of these significant differences for bank-level data, and the use of country aggregate data eliminates these effects.

the country-level parameter estimates as initial beliefs to estimate figures for μ_x and σ_x for individual banks within each country.

3.2 Estimating the cost parameters

For every bank, as stated above, we consider the sum of operating expenses and asset write-downs as the total cost. For the total cost figures of the banks, we use a basic structural time series model to facilitate the process of distinguishing between the cost of running the bank by a private investor c_e and by a government c_g. First, we identify the major additive outlier, a level shift or a slope change in the total cost time series to proxy the moment at which the cost structure changes. Having identified such an abnormal development, we estimate the private investors' cost of running the bank, as the upstream average total cost and the government's cost of running the bank represent the downstream average total cost.

In particular, to identify additive outliers, level shifts and slope changes, we estimate a local linear trend model by means of maximum likelihood and a Kalman filter with diffuse initialization. Smoothed disturbances from this model serve to identify these abnormal developments in the time series. The structural time series model we use is common in the literature, and we do not provide the details here (see Durbin and Koopman, 2012).

4. Results

This section presents our results for the sample of banks we consider in terms of the bail-out costs, the likelihood of bail-out, the bail-in cost compared with bail-out cost, the effect of macroprudential policies, the welfare value of the banks and the comparison of our results and the market capitalization of the banks.

4.1 Bail-out costs

Table 3 presents our country aggregate estimates for the bail-out costs using the four different scenarios we consider (fixed vs variable operational costs and level change vs no level change in the operational costs) and the two methods to determine the drift and volatility parameters (country vs individual bank). Table 3 only reports bail-out figures for the banks that were beneficiaries of public aid from 2008 to 2014.

In terms of the governmental aid figures, we present values from the European Commission (EC) and Eurostat. The EC aid figures consider recapitalizations, impaired asset measures, guarantees and other liquidity measures. The Eurostat figures for the Net Costs of Public Intervention merely take into account the public revenues and costs associated with the aid.

As becomes clear, although all the figures represent country aggregates, they differ significantly. This illustrates the difficulties in properly defining the boundaries of what are public bail-out costs in terms of their nature and the difficulties in determining the bail-out costs for individual financial institutions (e.g., see Fernandes et al. 2016). The estimation of bail-out costs represents a challenge that is widely recognized in the literature (Reinhart and Rogoff (2013) present a good discussion of the 'elusive concept of bail-out costs').

As the proper benchmark against which to compare our estimates, we will focus on the EC 'State Aid Used' figures. The EC state aid figures represent an effective comprehensive costs measure, including several aid instruments that in many cases differ more in form than in substance. Although the EC 'State Aid Used' figures ignore possible revenues, the short period considered makes it unlikely that the omission of the revenues might significantly distort the estimate of the real governmental aid costs. The only use of the recapitalization figures would be too narrow to compare with our bail-out estimates, since recapitalizations mainly address capital adequacy problems. Strict recapitalizations ignore impaired asset purchases and, notably, public guarantees. Similarly, the Eurostat 'Net Costs' figures only take into account the flows of costs and revenues, ignoring the guarantees and only taking them into account if they are called[18]. However, guarantees are a preferred method of public aid because they do not imply cash advances by the governments and do not compute for the public deficit calculations unless the guarantees are called.

Table 3: Estimated Bail-out costs for the banks that were recipients of state aid in the period 2008-2014

	Cyprus	Greece	Italy	Spain	Portugal
Scenario #1					
- Bank	43.9	224.8	96.4	279.3	183.7
- Country	40.2	201.6	96.1	279.0	154.7
Scenario #2					
- Bank	18.0	86.6	66.0	236.2	93.5
- Country	21.1	105.0	67.2	197.3	95.3
Scenario #3					
- Bank	23.3	161.9	80.1	225.7	163.6
- Country	18.3	133.5	79.9	209.5	133.3
Scenario #4					
- Bank	9.9	63.6	57.3	93.8	86.7
- Country	9.9	71.1	58.2	82.8	85.3
Range					
- Min	9.9	63.6	23.2	82.8	49.1
- Max	43.9	224.8	96.4	279.3	183.7
Governmental Aid Figures					
- State aid approved	9.5	160.6	106.2	544.2	73.5
- Recapitalization approved	3.5	59.6	25.8	174.3	34.8
- State aid used	6.3	115.8	97.4	186	38.8
- Recapitalization used	3.5	46.6	11.8	61.9	15.3
- Net Costs of Public Interv.	1.6	22.7	0.3	44.9	9.7

Notes: This table represents our estimates for the bail-out costs (ω=1) for all the banks in our sample that received state aid during the period 2008-2014 in EUR billions. For each country, it presents eight estimates combining the four Scenarios considered in terms of the behavior of the operational costs and the 2 estimations of the drift and volatility parameters. The state Aid and recapitalization figures, approved and used, are obtained from the European Commission State Aid Scoreboard for the period 2008 – 2014 and include recapitalizations, impaired asset measures, guarantees and other liquidity

[18] Given the current state of aversion of the general public to state aid to financial institutions and the fact that these figures are produced by the national governments, it is only natural to expect that in the Net Cost figures, the revenues may be somewhat inflated and the costs somewhat deflated.

measures. The net costs are obtained from Eurostat for the period 2008-2016 and include the net costs for governments to support financial institutions through different interventions.

In the comparison of our bail-out estimates with the EC 'State Aid Used' figures, we observe that for the cases of Greece and Spain, the figures of the state aid used fall within the range of bail-out costs estimated by our model. In the cases of Cyprus and Portugal, our bail-out figures overstate the state aid actually used, although the bank rescue case of Cyprus took more the form of a bail-in than a bail-out. If we consider the bail-in contributions, the privatization revenues and other measures (see EC, 2013), we find that the governmental aid in Cyprus falls within our estimated range for the bail-out costs. In the case of Italy, our estimates fall short of the state aid actually used, although it is close to the upper bound of our range.

Although the variability of both the bail-out cost estimates and of the effective governmental aid used is quite significant—therefore making comparisons difficult and somewhat speculative—we believe these results show that a structural approach to the estimation of bank rescue costs is able to generate reasonable estimates, given that the appropriate modeling assumptions are made.

With respect to the assumptions made about the behavior of the operational costs (the different scenarios) and about the estimates of the drift and volatility parameters (individual bank vs country) we find considerable heterogeneity in our bail-out estimates. In terms of the cost scenarios, our results reveal no clear dominance of one single scenario, although we apply the methodology to markets that present important geographical and cultural proximities. Whether we consider all the combinations of bail-out estimates or the averages between individual bank and country estimates for the drift and volatility parameters, we observe that Scenario 1 presents the closest estimate for Italy, Scenario 2 for Greece and Scenario 4 for Cyprus and Portugal. In light of our assumptions, these results seem to indicate that Italian banks may have higher operational leverage with respect to the other countries in our sample[19] and that during our sampling period, there were important cost shifts in the cases of Italy, Spain and Greece.

Considering all the combinations of bail-out estimates, the best estimates are obtained with the country data in what concerns the estimation of the drift and volatility parameters, with the exception of Italy. However, in the Italian case, the difference in results between the two alternative ways of estimating the drift and volatility parameters is absolutely marginal. When we consider averages taking into account the four different scenarios, we observe that the country estimates are the closest to the 'State Aid Used' figures for all the countries.

The proximity of the bail-out estimates is remarkable for the Italian case in terms of the use of individual bank and country estimates for the drift and volatility parameters. In all the other countries, although close, the differences are more important. A possible explanation for these differences is the fact that during our sampling period, Italy had the lowest number of banks that were beneficiaries of public aid. In other words, it is possible that in countries in which a significant number of banks resort to public aid, there is more pressure on the supervisor and on the banks themselves to recognize losses and impaired assets, therefore creating more volatility in the earnings and costs and more heterogeneity in the results among the different entities. Countries in which few banks resort to public aid will therefore present more-stable earnings and costs and greater homogeneity in estimates.

Beforehand, we expected to find similarities between the results in terms of the size of the banking market and in terms of geographical proximity; however, our results contravene this

[19] We have tried to confirm this hypothesis through an analysis of the degree of Operating Leverage considering all the banks in our sample. However, the high variability of the revenues and earnings during our sampling period invalidates this type of analysis.

hypothesis. In terms of market size, we find important differences in the best scenarios for the cases of Portugal and Greece and for the cases of Spain and Italy (in both cases, the markets have a similar size). In terms of geographical proximity, we find important differences in the best scenarios for the cases of Portugal and Spain and for the cases of Cyprus and Greece (in both cases, there is an important cultural and geographical proximity). Moreover, although we depart from a geographically and culturally homogeneous sample of countries, our results show that we do not have a one-model-fits-all solution. In the following subsection, we analyze the accuracy of a structural model in terms of anticipating the need for a public rescue.

4.2 Likelihood of bail-outs

In addition to the estimates of the bail-out costs we discussed previously, our structural model also produces estimates of the expected time to bail-out and of the probability of bail-out. In this section, we compare the accuracy of the model in estimating the number of entities that are in need of a bail-out with the number of entities that were effectively beneficiaries of public aid during our sampling period. We define three different categories for the banks in terms of the likelihood of being bailed out. The Immediate need of bail-out includes all banks for which the expected time to bail-out is smaller than zero, and consequently, the probability of bail-out is 100%. The category 'Serious Risk' includes all banks for which the expected time to bail-out is less than five years. Finally, the safe category includes all banks for which the expected time to bail-out exceeds five years.

Table 4 presents the distribution of the banks in our sample among the three categories we defined and considering four different scenarios (fixed vs variable operational costs and level change vs no level change in the operational costs) and the two methods to determine the drift and volatility parameters (country vs individual bank).

Table 4: Likelihood of a bank bail-out

	Cyprus		Greece		Italy		Spain		Portugal	
	Bnk	Cnty	Bnk	Cnty	Bnk	Cnty	Bnk	Cnty	Bnk	Cnty
Scenario #1										
- Immediate need	0	0	1	1	10	10	7	1	3	4
- Serious risk	0	0	2	1	2	1	0	1	2	1
- Safe	2	2	1	2	3	4	5	10	1	1
Scenario #2										
- Immediate need	0	0	0	0	3	3	7	0	0	1
- Serious risk	0	0	0	0	4	4	0	0	4	1
- Safe	2	2	4	4	8	8	5	12	2	4
Scenario #3										
- Immediate need	0	0	1	1	10	10	6	3	3	5
- Serious risk	0	0	3	3	2	2	1	0	3	1
- Safe	2	2	0	0	3	3	5	9	0	0
Scenario #4										
- Immediate need	0	0	0	0	3	5	6	1	0	1
- Serious risk	0	0	0	0	5	2	0	0	5	2
- Safe	2	2	4	4	7	8	6	11	1	3
Aided banks	2	2	4	4	1	1	3	3	4	4

Notes: this table presents the classification of all the banks in our sample in terms of the likelihood of being bailed out. The last row shows the number of banks that have received public aid in the period

2008-2014. The Immediate need of bail-out includes all banks for which the $x<x_L \rightarrow \vartheta_L<0$ and $P>100\%$. Serious risk includes all banks for which $\vartheta_L \leq 5$ years, and safe includes all banks for which $\vartheta_L>5$ years.

Contrary to the bail-out cost estimates, in which there was a considerable difference between the scenarios, and even considering that we perform our analysis using a classification of intervals in the estimation of the likelihood of default, we now observe that the differences between scenarios are narrower.

As becomes clear with the case of Cyprus, our model does not identify all the banks that were bailed out, and it classifies banks that were bailed out or beneficiaries of public aid as being in immediate need or serious risk. Given the short time span of our data series and the fact that they cover a period of financial crisis, this result is not entirely unexpected.

In terms of the scenarios, we observe that Scenario 3 is the most accurate for the cases of Greece and Portugal, Scenario 2 for the case of Italy and Scenario 1 for the case of Spain. In none of these cases do the scenarios match the results of the bail-out costs analyzed previously. There is a fundamental difference between the two analyses. In terms of the bail-out costs, they are estimated at the moment the bail-out occurs, and they were calculated only for the banks we identified as being the recipients of public aid. When calculating the likelihood of bail-out, we use the entire sample of banks.

Naturally, we observe that the scenarios that consider fixed operational costs, such as Scenarios 1 and 3, generate an overall higher likelihood of bail-out. We also observe that when the level change in costs is considered (Scenarios 1 and 2), our estimates of the number of banks in immediate or serious risk of bail-out are smaller relative to the scenarios in which we consider no level change. The explanation is that when the costs are constant, the average operational costs while the bank is run by the private equity holders are higher than when we consider the level change. This higher operational cost naturally leads the bank into bail-out earlier. If we analyze the absolute differences between the number of banks that require or are in serious risk of requiring bail-out with and without the cost level change, we see no difference in terms of accuracy. Apparently, both assumptions are reasonable. However, it is very difficult to identify which assumption is more appropriate for each country before we perform the analysis.

Considering all the different scenarios and the sum of the categories 'Immediate need' and 'Serious risk' and its comparison with the number of aided banks, we observe that the best results are obtained with the bank estimations for the cases of Greece, Spain and Portugal. Italy once again presents very similar results when using bank or country estimates, although country estimates produce marginally more-accurate results. In the case of Cyprus, there is no difference between using bank or country estimates. Overall, the differences between using country or bank estimates are not as significant as in the case of the estimation of the bail-out costs.

4.3 Bail-out vs bail-in costs

In the aftermath of the financial crisis and of the massive bail-out programs implemented, there was public outcry and the recognition that bail-outs, in many instances, translated into private gains and public losses, increased moral hazard and rewarding excessive risk-taking behavior. Since then, we have witnessed a shift in European Authorities from favoring a bail-out to clearly favoring a bail-in. In the last two EU bail-outs (the second bail-out of Greece in 2012 and the bail-out of Cyprus in 2013), there were already losses imposed on private investors, effectively marking the change from a strict bail-out to a bail-in policy. With Directive 2014/59/EU, this shift from a bail-out to a bail-in policy was effectively put into law.

In this section, we analyze the effects of a change from a strict bail-out to a bail-in policy on the rescue costs for a government. Naturally, in our bail-in analysis, we are unable to incorporate possible responses from corporate lenders in terms of an increase in the credit spreads, as we have done in subsection 2.5.4. Therefore, the main result in this section is the actual rescue cost, since the expected time to the rescue and the rescue probabilities remain unchanged with respect to the bail-out case.

Table 5 presents the bail-out and bail-in costs for our subsample of banks that were recipients of public aid during the sampling period. Contrary to the previous results, we only consider the case of Scenario 4 of variable operational costs with no level change and with country estimates for the drift and volatility parameters[20]. In the case of bail-in, we consider several alternatives, ranging from a loss of 25% imposed on corporate lenders (ω=0.75) to a case in which they may lose all value (ω=0.0).

Table 5: Bail-out vs bail-in costs

	Cyprus	Greece	Italy	Spain	Portugal
Bail-out (ω=1)					
- Country est.	9.9	71.1	58.2	82.8	85.3
- D/E	0.3	1.1	444.2	4.1	112.0
Bail-in (ω=0.75)					
- Country est.	9.5	67.3	44.4	69.7	68.3
- %Δ to Bail-out	-3.8%	-5.4%	-23.6%	-15.8%	-19.9%
Bail-in (ω=0.5)					
- Country est.	9.1	63.5	30.7	56.6	51.3
- %Δ to Bail-out	-7.5%	-10.7%	-47.2%	-31.7%	-39.9%
Bail-in (ω=0.25)					
- Country est.	8.7	59.7	17.0	43.5	34.3
- %Δ to Bail-out	-11.3%	-16.1%	-70.8%	-47.5%	-59.8%
Bail in (ω=0.0)					
- Country est.	8.4	55.9	3.2	30.4	17.3
- %Δ to Bail-out	-15.0%	-21.4%	-94.4%	-63.3%	-79.7%

Notes: This table presents the values for the bail-out considering Scenario 4 and with the drift and volatility parameters estimated with country data. It also presents for the same scenario the cases of bail-in, in which ω=0.75, 0.5, 0.25 and 0.0. No change in the credit spreads of debt is considered. The D/E ratio is calculated as the value of corporate debt estimated using equation (19) over the value of equity calculated using equation (19), in both cases assuming that the operational costs are variable.

The results of Table 5 show a clear reduction in the rescue costs associated with the move from a bail-out to a bail-in policy. As the bail-in parameter ω decreases, we observe a decrease in the rescue costs for the government in all the countries. However, we also observe that the reduction in the rescue costs is not homogeneous across countries, with Italy, Portugal and Spain presenting the larger reduction in rescue costs. Since the bail-in only affects the corporate debtholders, leaving depositors unaffected, we expect that banks with a greater financial leverage are those more affected by the move from a bail-out to a bail-in policy.

The differences in the reduction in costs are related to the use of corporate debt. Naturally, countries in which the banks use more corporate debt have higher gains in changing from a bail-

[20] Although we only report one scenario, the qualitative nature of the results and, most importantly, the size of the changes in the rescue costs for the different bail-in alternatives is in all aspects similar for all scenarios and for the case of individual bank estimates for the drift and volatility parameters. The results for all scenarios are available to the reader upon request.

out to a bail-in policy. This becomes clear when comparing the leverage ratios with the relative change in the rescue costs when moving from a bail-out to a bail-in policy.

4.4 Macroprudential Policies

In the previous section, we showed how a bail-in is able to reduce the rescue costs of a government relative to a bail-out solution. However, the implementation of macro- and micro-prudential policies can achieve such an objective and maybe even avoid a bank rescue.

In this section, we show how our model can be used to anticipate the outcomes of such policies. Our approach is as follows. From the static analysis of section 2.5, we are able to determine the effects of different macro- and micro-prudential policies by examining how the implementation of these policies affects our model parameters.

In a simple exercise, we simulate the bank results following the sale of their more-volatile business lines to concentrate on their core interest-related activity. We conduct this exercise with the Greek banking sector, as represented in this paper by its four largest banks.

To quantify the effects of this policy on the Greek banks, we assume that as a result of its implementation, the banks' total income becomes simply net interest income, as explained in section 3. This will translate into a significant decrease in risk σ and a non-negligible increase in drift μ for all four banks vis-à-vis the case in which they were engaged in alternative investment banking activities. Additionally, their operating costs are scaled down by a factor that results from the ratio of mean total income to mean net interest income over the sample period. This ratio represents, on average, 80%.

As result of the implementation of this policy, we estimate that, as expected, the rescue costs decrease substantially. For example, considering scenario 3 and using individual bank data to determine the drift and volatility parameters, the bail-out costs drop from 161.9 EUR billions, as presented in Table 3, to 96.8 EUR billions. However, this significant drop is not homogenous across scenarios. For instance, in Scenario 4, using country data to determine the drift and volatility parameters, our estimate of the reduction of the bail-out costs is considerably smaller. The bail-out costs drop from 71.1 EUR billions, as presented in Table 3, to 58.8 EUR billions.[21]

These results are consistent with the expected results of a bank disinvestment of non-core activities and thus constitute a clear example of possible applications of our model as a testing ground for a different array of macro-prudential policy measures.

4.5 Welfare value of the banks

We consider a welfare measure for the value of the banks consisting of the sum of the value accruing to its investors (equity holders and debtholders) and the value accruing to the government. The welfare values we present in Table 6 take into account all the banks in our sample and consider only the case of a bail-out.

[21] As before, although we do not report all scenario results, the qualitative nature of the results is in all aspects similar for all scenarios. The results for all scenarios are available to the reader upon request.

Table 6: Welfare value of the banks

	Cyprus	Greece	Italy	Spain	Portugal
Scenario #1					
Bank	-32.0	-179.3	-893.6	-107.9	-139.0
Country	-27.4	-147.4	-840.1	-493.5	-116.5
Scenario #2					
Bank	-4.2	-15.6	-114.2	157.4	-8.1
Country	-3.9	-18.3	-131.3	78.7	-12.3
Scenario #3					
Bank	-15.9	-141.6	-571.3	46.6	-120.9
Country	-12.0	-115.7	-516.2	-253.2	-98.2
Scenario #4					
Bank	1.6	-9.2	10.1	237.3	-5.7
Country	2.1	-13.8	6.9	171.2	-9.5
Range					
Min	-32.0	-179.3	-893.6	-493.5	-139.0
Max	2.1	-9.2	10.1	237.3	-5.7
Average (a)	-11.5	-80.1	-381.2	-20.4	-63.8
Market size (b)	91	398	4.022	2.973	470
Avg. Relative (a)/(b)	-12.6%	-20.1%	-9.5%	-0.7%	-13.6%

Notes: this table presents the sum of the welfare value of all the banks in our sample for the case of a governmental bail-out. The welfare value is computed as the sum of the values accruing to equity holders, debtholders and the government. The range and the average are calculated across the different scenarios and country and bank estimates. The Market Size represents the Total Value of bank assets in December 2014, as reported by the ECB. Values are in Millions of Euro.

Our results show that in many of the cases, even when the bank has not been bailed-out, the welfare value is negative. Knowing that the values of equity and debt have a lower bound of zero, this means that the value for Government, assuming that it may have to bail out banks in the future, is negative. In simple words, the expected present value of the future bail-out costs (taking into account its likelihood) far exceed the present value of the perpetual stream of revenue that the Government collects as corporate taxes.

Although our total averages across the different scenarios are negative for all the countries, a more detailed analysis by country shows that Greece, Portugal and Cyprus obtain the worst results, followed by Italy and Spain. These results should not be entirely surprising, and the rankings are perfectly in line with the results of the 2014 Banking Stress Tests, in which Greece, Cyprus, Portugal and Italy present some of the largest expected shortfalls of Tier 1 capital ratios in the adverse scenarios tested. For all these countries, the adverse scenario presents an impact that largely exceeds the median of the Single Supervisory Mechanism (see EBA, 2014). The case of Spain, on the other hand, presents one of the smallest expected shortfalls in the adverse scenario of the EU countries, being far below the median of the Single Supervisory Mechanism.

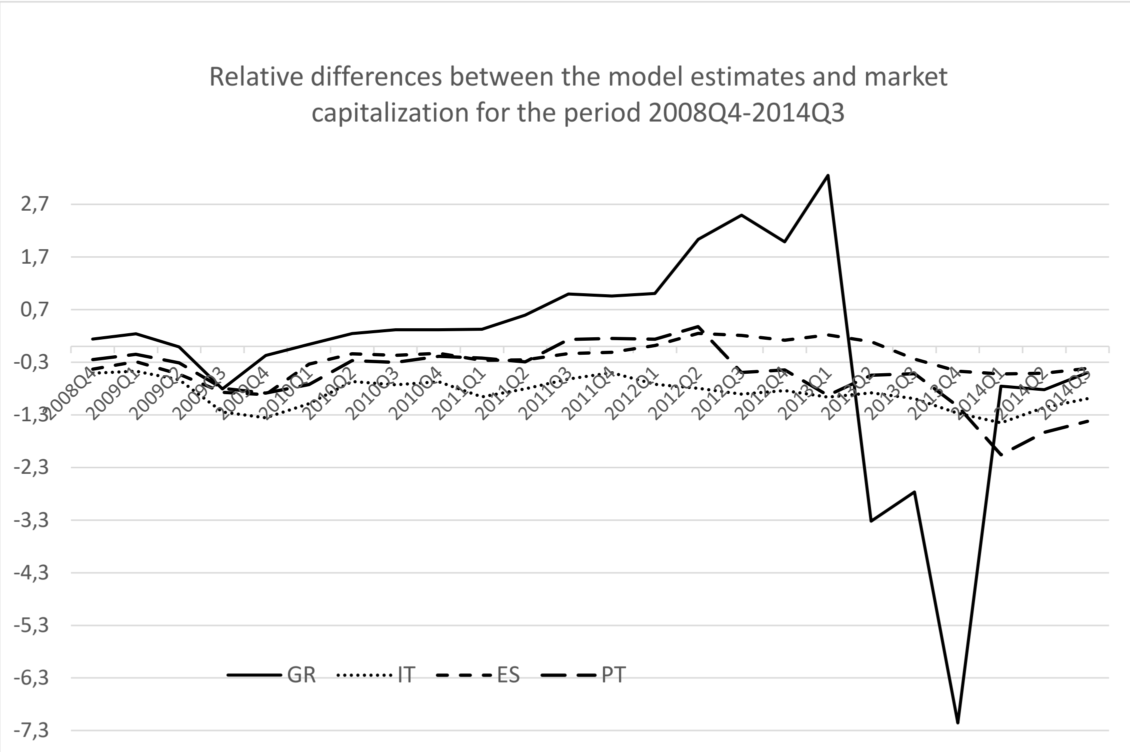
These results show that regardless of the value they create, banks also consume a lot of value, and given a system of public bail-outs, the government ends up absorbing the shortfall. Given our previous results in terms of the savings generated with the change from a bail-out to a bail-in system, we can conclude that in terms of welfare, this change will naturally benefit more those countries that currently present the most-negative welfare values, such as Greece, Portugal, Cyprus and Italy. In the Spanish case, the benefits in terms of welfare will not be as significant.

4.6 Market Capitalization

In this section, we compare our estimates for the value of equity with the equity market values. Our estimates are calculated on a quarterly basis based on the changes in earnings and operational costs. We assume that the operational costs are variable and that there is no level change (Scenario 4).

Figure 2 presents the relative differences between our estimates and the market values between the 4th quarter of 2008 and the 3rd quarter of 2014.

Figure 2: Differences in equity estimates and market values



Notes: this figure presents the differences between our estimates for the equity value and the market capitalization of the banks between the 4th quarter of 2008 and the 3rd quarter of 2014. In terms of assumptions, we consider Scenario 4 and include all banks for which we could obtain share prices.

Figure 2 shows important differences between our equity value estimates and the market values of equity. Naturally, we expected to find important differences, since market values react instantly to the arrival of new information, whereas our model estimates only capture these changes when they affect the earnings figures.

The main differences between our estimates and the market values are essentially threefold. First, there are the differences in estimates for the 3rd and 4th quarters of 2009, in which our estimates undervalue equity for all the banks in the sample. Second, there is the case of Greece, in which our estimates significantly overvalue equity from the 2nd quarter of 2010 to the 1st quarter of 2013 and significantly undervalue equity from the 2nd quarter of 2013 to the 4th quarter of 2014. Finally, we observe that the model estimates for the value of equity are generally below the market value of equity. In simple words, the model is underestimating the equity value.

The first difference concerning the 3rd quarter of 2009 coincides with the ECB announcement of a new provision of liquidity to European banks and similar announcements by national governments providing guarantees to bank lenders and providing banks with capital resources.

The second difference concerning the overvaluation of equity for Greek banks between the 2nd quarter of 2010 and the 1st quarter of 2013 coincides with the worst period of the Greek crisis, during which there was considerable economic, financial and political instability. During this period, Greece was subject to two bail-out packages for its economy, and seven austerity packages were passed through parliament. At the same time, there were four different governments and at least two important government reshuffles, there was a succession of credit downgrades, and there were major public protests and riots. In this context, it is not surprising that our model overestimates the value of equity, since the market values rapidly incorporated each of the aforementioned items of bad news. In the period ranging from the 2nd quarter of 2013 to the 4th quarter of 2014, our model underestimates the equity value of Greek banks. During this period, there was an important recapitalization of the Greek banks performed by the Hellenic Financial Stability Fund totaling 28 billion euros. This represented good news that was rapidly incorporated into the market valuations of the equity of Greek banks.

Finally, there may be different explanations for the general undervaluation of equity by our model. One explanation concerns the data used by the model and the fact that the accuracy of this data is sometimes questionable (see FT, 2017, and The Economist, 2017). The timeliness of the data is sometimes not perfect, and it is well known that insiders have some control over the flows of information (see Ang and Chen, 2002, and Acharaya et al., 2011). Another explanation concerns the model itself and some of its assumptions. To proxy cash flows, we need to include asset write downs, but by doing so, there is the risk that we may be incorporating as a change in drift what is in reality a level change. The reasonableness of the model results regarding the rescue costs and rescue probabilities do not seem to indicate such bias, but it is a real possibility.

5. Conclusions

The structural model presented in this paper serves primarily to predict and estimate the future costs of a bank rescue and the likelihood that the rescue will take place. It allows a bail-out or a bail-in solution, and it can predict the impact of different macro- and micro-prudential policies, as our example of the Greek banks shows.

Regardless of the reasonability of our results, the model also highlights the main challenges of structural modeling, since our results are very sensitive to both the assumptions made regarding the behavior of the operational costs of the banks and to the origin of the results used for the volatility and drift parameters. This task is not simple, since in departing from reasonable expectations of homogeneity of the southern European countries analyzed in terms of their cultural and geographical proximity, we observe important differences that do not correspond to our initial beliefs of proximity. Regarding the assumptions made, we have found that neither geographical or cultural proximity nor even market size are able to explain the differences of our results.

The current context of political uncertainty, and our very own estimates of expected times to bail-out for several EU financial institutions, make it unlikely that the issue of bank failures will disappear from the public agenda in the near future. This model represents a tool to address the problems of bank rescues, but it is a simple first step. Although a social welfare measure has been introduced in this paper, future developments of this model may consider a broader social welfare measure that would allow us to address an important question still on the public agenda, which is the question of the size of the financial industry, of the value it generates and of the value it consumes.

APPENDIX

Appendix 1: Proofs of Propositions

Proof of Proposition 1

The expressions (17)-(19) for the values of $E(x)$, $G(x)$ and $D(x)$, respectively, are obtained by replacing in expression (11) the corresponding values for *a* and *b* from Table 1 and the corresponding values for L_A from the value-matching boundaries (12)-(14) for each of the claims valued.

Proof of proposition 2

The value expression for $R(x_L)$ is simply the value-matching boundary condition (13). The bank rescue trigger is obtained from the optimal decision of equity holders to abandon the bank. Equity holders basically choose the optimal moment to change between two value functions,

$$E(x)=\left(\frac{x}{\delta}-\frac{c_E+i}{r}\right)(1-\tau)-\left(\frac{x_L}{\delta}-\frac{c_E+i}{r}\right)(1-\tau)\left(\frac{x}{x_L}\right)^{\beta_2} \tag{A.1}$$

and

$$E(x)=0 \tag{A.2}$$

when determining the value function (A.1), the value matching condition (12) insures that at x_L there will be no discontinuity between (A.1) and (A.2). However, for x_L to be optimal, in the sense that it maximizes the present value of equity, we must also have no discontinuity between the slopes of (A.1) and (A.2) at x_L. If the slopes are different at x_L, the function with a higher slope would obviously be better, and a different x_L should optimally be determined. Thus, we have a smooth pasting or optimality condition ensuring the equality of the slopes at x_L,

$$\left.\frac{\partial E(x,x>x_L)}{\partial x}\right|_{x=x_L}=\left.\frac{\partial E(x,x<x_L)}{\partial x}\right|_{x=x_L} \tag{A.3}$$

since

$$\left.\frac{\partial E(x,x<x_L)}{\partial x}\right|_{x=x_L}=0$$

we have

$$\left.\frac{\partial E(x,x>x_L)}{\partial x}\right|_{x=x_L}=0 \,. \tag{A.4}$$

For simplicity, expression (A.4) is usually presented as expression (20). Once (A.4) is derived with respect to x, and x is replaced for x_L, isolating x_L yields the analytical explicit equation (22).

Following Shackleton and Wajokowski (2002), we convert the abandonment trigger x_L into an expected time to exercise of abandonment. The probabilities of bank rescue under the real world measure $\mathbb{P}$ were calculated following Thijssen (2010).

Appendix 2: Estimating the drift and volatility parameters

We recall that total income x follows a geometric Brownian motion

$$dx = \mu_x x dt + \sigma_x x dz$$

where μ_x is the instantaneous growth rate of the total income or drift, σ_x is the standard deviation or risk, and dz is the increment of a standard Wiener process. Thus, the distribution of x's logarithm differences $y_t = \log\left(\frac{x_t}{x_{t-1}}\right)$ is Normal

$$y_t \sim \mathrm{N}(m, s^2)$$

for $t = 2, \ldots, T$, with $T = 27$ in our analysis, where m is the mean and s^2 is the variance.

After Richardson (2011), we assume the following prior distributions, Gaussian for the mean m and Gamma for $\theta^2 = \frac{1}{s^2}$, the precision under which our prior beliefs about these parameters are independent

$$m \sim \mathrm{N}\left(m_0, \frac{1}{\tau_0^2}\right)$$

and

$$\theta^2 \sim \mathrm{G}\left(\frac{v_0}{2}, \frac{v_0}{2\theta_0^2}\right)$$

where m_0 and $\tau_0, v_0, \theta_0 > 0$ are constants specifying the above distributions.

These constants let us now introduce our initial beliefs about drift μ_x and risk σ_x parameters into the prior distributions for m and θ^2 in the following way.

From the geometric Brownian motion assumption about x, we know that $m = \left(\mu_x - \frac{1}{2}\,\sigma_x^2\right)\Delta$ and $s^2 = \sigma_x^2\Delta$, where Δ is the step. Hence, $\frac{1}{\sigma_x^2}$ is assumed to follow a Gamma distribution of the form

$$\frac{1}{\sigma_x^2} \sim \mathrm{G}\left(\frac{v_0}{2}, \frac{v_0}{2\Delta\theta_0^2}\right)$$

and μ_x a Gaussian distribution of the form

$$\mu_x \sim \mathrm{N}\left(m_0 + \frac{1}{2}[p(\sigma)]^2, \frac{1}{\Delta^2\tau_0^2}\right)$$

where we substitute σ_x with $p(\sigma_x)$, which is our prior belief about the risk parameter. We further denote $p(\mu_x)$ as our prior belief about the drift parameter. In addition, $c(\mu_x)$ and $c\left(\frac{1}{\sigma_x^2}\right)$ represent how confident we are about our initial beliefs, in this case, regarding μ_x and $\frac{1}{\sigma_x^2}$.

Because we know that

$$E(\mu_x) = m_0 + \frac{1}{2}[p(\sigma_x)]^2$$

$$Var(\mu_x) = \frac{1}{\Delta^2 \tau_0^2}$$

$$E\left(\frac{1}{\sigma_x^2}\right) = \theta_0^2 \Delta$$

$$Var\left(\frac{1}{\sigma_x^2}\right) = \frac{2\Delta^2 \theta_0^4}{v_0}$$

we assign $p(\mu_x) = E(\mu_x)$, $p(\sigma_x) = E(\sigma_x)$, $c(\mu_x) = \sqrt{Var(\mu_x)}$ and $c\left(\frac{1}{\sigma_x^2}\right) = \sqrt{Var\left(\frac{1}{\sigma_x^2}\right)}$ to immediately arrive at

$$m_0 = p(\mu_x) - \frac{1}{2}[p(\sigma_x)]^2$$

$$\tau_0 = \frac{1}{\Delta c(\mu_x)}$$

However, we still need a formula for $\theta_0$and v_0. To that end, we assume $E\left(\frac{1}{\sigma_x^2}\right) = \frac{1}{[p(\sigma_x)]^2}$, which leads to

$$\theta_0 = \frac{1}{p(\sigma_x)\sqrt{\Delta}}$$

$$v_0 = \frac{2[p(\sigma_x)]^2}{\left[c\left(\frac{1}{\sigma_x^2}\right)\right]^2}$$

With the above formulas for m_0, τ_0, θ_0 and v_0 at hand, which allow us to introduce our initial beliefs about drift and risk as well as how confident we are about them, we can proceed with a Gibbs sampler, as in Richardson (2011), to eventually obtain a sequence of dependent samples of m and θ^2.

From here, we are only two steps away from obtaining posterior estimates of μ and σ, which are $\sigma_x = \sqrt{\frac{1}{\Delta\theta^2}}$ and $\mu_x = \frac{m}{\Delta} + \frac{\sigma_x^2}{2}$, in that order.

In the first estimation, at the country level, we set our prior beliefs at $p(\mu_x) = 0\%$ and $p(\sigma_x) = 20\%$, whereas the confidence levels are kept at $c(\mu_x) = 0.1$ and $c\left(\frac{1}{\sigma_x^2}\right) = 5$, including in the second estimation. Therein, at the individual bank level, we vary $p(\mu_x)$ and $p(\sigma_x)$ based on corresponding results from the first estimation for each country.

REFERENCES

Acharaya, V., DeMarzo, P. and I. Kremer, 2011. Endogenous Information Flows and the Clustering of Announcements. American Economic Review 101, 2955–2979.

Ang, A. and J. Chen, 2002. Asymmetric Correlations of Equity Portfolios. Journal of financial Economics, 63, 443–94.

Bali, Turan G., Hao Zhou, 2011. Risk, Uncertainty, and Expected Returns. Unpublished Georgetown University working paper.

Berg, A., Pattillo, C., 1999. Predicting currency crises: the indicators approach and an alternative. Journal of International Money and Finance 18, 561-586.

Boyd, J.H., Daley, L., Runkle D.E., 1993. The seasonality of interim discretionary accruals: the case of loan-loss provisions and chargeoffs in the banking industry. Fed of Minneapolis working paper 515.

Boyd, J., Kwak, S., Smith B., 2005. The Real Output Losses Associated with Modern Banking Crises. Journal of Money, Credit and Banking 37, 977-999.

Bussiere, M. and M. Fratzscher 2006. Towards a new early warning system of financial crises. Journal of International Money and Finance 25, 953-973.

Calvo, G., 2005. Varieties of Capital-Market Crises. In G. Calvo Emerging Capital Markets in Turmoil: Bad Luck or Bad Policy?. Cambridge, MA: MIT Press.

Childs, P., Mauer, D. and S. Ott, 2005. Interactions of corporate financing and investment decisions: The effects of agency conflicts. Journal of Financial Economics 76, 667-690.

Davis, E. and D. Karim, 2008. Comparing early warning systems for banking crises. Journal of Financial Stability 4, 89-120

Dermine, J., Schoenmaker D., 2010. In Banking, Is Small Beautiful?. Financial Markets, Institutions & Instruments 19, 1-19.

Diamond, D., Dybvig P. H., 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91, 401-419.

Directive 2014/59/EU of the European Parliament and of the Council of 15 May 2014, European Comission.

Dixit, R., Pindyck, R., 1994. Investment under uncertainty. Princeton University Press.

Dubrin, J., Koopman, S. J., 2012. Time Series Analysis by State Space Methods. Oxford University Press.

Eschenbach, F., Schuknecht L., 2002. The fiscal costs of financial instability revisited. European Central Bank Working Paper n. 191.

Frankel, J., Rose, A., 1996. Currency crashes in emerging markets: an empirical treatment. Journal of International Economics 41, 351-366.

Friedman, M., Schwartz A., 1963. A Monetary History of the United States, 1867-1960. Princeton University Press, New Jersey: USA.

FT, 2017. Spain launches criminal probe into ill-fated Bankia flotation, The Financial Times https://www.ft.com/content/9647fdda-f1f6-11e6-8758-6876151821a6

Goldstein, M., Kaminsky, G., Reinhart, C., 2000. Assessing Financial Vulnerability: An Early Warning System for Emerging Markets. Institute for International Economics, Washington, DC.

Goldstein, R., N. Ju, and H. Leland, 2001. An EBIT-based model of dynamic capital structure. Journal of Business 74, 483-512.

Gordon, M.J. and E. Shapiro, 1956. “Capital equipment analysis: the required rate of profit”, Management Science 3, 102–110.

Gropp, R., Hakenes, H., Schnabel I., 2011. Competition, Risk-shifting, and Public Bail-out Policies. Review of Financial Studies 24, 2084-2120.

Guriev, S., Kolotilin, A., Sonin, K., 2011. Determinants of nationalization in the oil sector: A theory and evidence from panel data. Journal of Law, Economics, and Organization 27, 301-323.

Hakenes, H. and Schnabel I., 2010. Credit risk transfer and bank competition. Journal of Financial Intermediation 19, 308-332.

Hart, O. and Zingales L., 2008. Economists Have Abandoned Principle. Wall Street Journal.

Heider, F., Hoerova, M., Holthausen C., 2008. Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk. Working Paper European Central Bank.

Hoff, P. D., 2009. A First Course in Bayesian Statistical Methods. Springer.

Johnson, S., 2009. The Quiet Coup. The Atlantic.

Kaminsky, G., Lizondo, S., Reinhart, C., 1998. Leading indicators of currency crises. IMF Staff Papers 45, 1-48.

Kaminsky, G., Reinhart C., 1999. The Twin Crises: The Causes of Banking and Balance of Payments Problems. American Economic Review 89, 473-500.

Korteweg, A., 2010. The Net Benefits to Leverage. The Journal of Finance 65, 2137-2170.

Laajimi, S., 2012. Structural Credit Risk Models: A Review. Insurance and Risk Management 80, 53-93.

Leland, H.E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49, 1213-1252.

LeRoy, S.F., Porter, R.D., 1981. The present value relation: tests based on implied variance bounds. Econometrica 49, 555-74.

Megginson, W., 2005. The Financial Economics of Privatization. Oxford University Press.

Merton, R. (1974). On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance 29, 449-470.

Philippon, T., Schnabl P., 2009. Efficient Recapitalization. NBER working Paper 14929.

Raff, H., 1992. A model of expropriation with asymmetric information. Journal of International Economics 33, 242-265.

Reinhart, C., Rogoff S., 2013. Banking crises: An equal opportunity menace. Journal of Banking & Finance 37, 4557–4573.

Richardson, T., 2011. Bayesian inference for normal mean and precision under independent priors. University of Washington, Department of Statistics, mimeo.

Shackleton, M., Wojakowski R., 2002. The expected return and exercise time of Merton-style real options. Journal of Business Finance and Accounting 29, 541-555.

Shleifer, A., 1998. State versus private ownership. The Journal of Economic Perspectives 12, 133-150.

Stroebel, J., van Benthem, A., 2010. Resource extraction contracts under threat of expropriation: Theory and evidence. USAEE/IAEE Working Paper, 1-38.

Schwartz, E., Trolle, A., 2010. Pricing expropriation risk in natural resource contracts: A real options approach. In Hogan, W. and Sturzenegger, F. (Eds). The Natural Resources Trap: Private Investment Without Public Commitment, 263-288, MIT Press.

Stiglitz, J., 2000. Economics of the Public Sector (Thrid Ed.). W.W. Norton & Company.

Sundaresan, S. (2013). A Review of Merton's Model of the Firm's Capital Structure with its Wide Applications. Annual Review of Financial Economics 5, 21-41.

The Economist, 2017. Banco Popular fails and is bought by Santander, The Economist, http://www.economist.com/news/finance-and-economics/21723143-european-bank-crises-go-was-orderly-one-banco-popular-fails-and

Thijssen, J., 2010. Irreversible Investment and Discounting: an Arbitrage Pricing Approach. Annals of Finance 6, 295-315.

Trueman B., Titman, S. , 1988. Studies on Management's Ability and Incentives to Affect the Timing and Magnitude of Accounting Accruals. Journal of Accounting Research 26, 127-139.

Veronesi, P., Zingales L., 2010. Paulson's gift. Journal of Financial Economics 97, 339–368.

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